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The Arts of Percolation

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I never like to be in a crowd. It is terrifying how people coming together lose their identity to become mere items in a mob. Shakespeare knew this, for in *Julius Caesar* (Act III Scene II) Mark Anthony talks to the Roman people and raises such a mop that revenge the death of Caesar. The formation of mob is a percolation.

Percolation is a study of connection of components which appear in two states. Without losses of generality let us call these latter *yes* and *no*. Neighbour components having the same state form a cluster, which is sometimes called *animal*. As components change their state, the animals they form change their shape and size. These animals or clusters fall apart into smaller clusters, or merge with one another to form a larger one. They have a life and lifetime of their own.

Thinking individuals coming together form a group of people, a gathering. But a mob is a group of people in the same polarised state. However euphonious a word one may use, a mob is an animal. Individuality demands that no two persons are the same, and therefore differences are respected. But the criterion of mobilisation reduces individuality into binary states, one is either for or against that physical formation justly called a *mob*. Whether we like it or not, call it a witchcraft if you like, or describe it as black, but the fact remains the skill of mass mobilising is an art. Shakespeare obviously knows everything about it when he made his Marcus Antonius say, 'I come to bury Caesar, not to praise him. The evil that men do lives after them, The good is oft interred with their bones; So let it be with Caesar,' for indeed he was only there for the sole purpose of burying his dead friend Caesar. It was no fault of his that precisely by the end of his talk the Roman people mobilised to revenge Caesar's ghost. And those people led by Brutus, who together murdered Caesar, and who were all this time sitting by, did not notice anything wrong until it was already too late.

Cluster formation could be engineered. But the result would be in an excited, unstable state. Moreover, such clusters are rarely large

enough to span the whole space, especially when the network has high connectivity and is distributed. It is very difficult to engineer a mob, for instance, over a large area. There is a difference between a mob and the silent majority in democracy, in that the latter is a largest, space-spanning cluster called *percolating cluster*. Whereas it is possible to temporarily induce the former locally, the latter is invariably based on a pacific, that is peaceful process that goes on within a society which maintains a proper distance among the thinking individuals.

In percolation one needs to distinguish between the gradual process in the background and the abrupt phenomena that one notices on the outside. In this case the potential is everything, and the power nothing but a consequence of it. Thus the power of a good king is never in himself but in the people, and is rooted in the dedication and prudence they have seen in him in his position. Similarly the power of the media is also not in itself but in the people. Both blames for a thing which triggers a crisis, and credits for an immediate factor that heralds prosperity, tend to be an exaggeration. Take for instance a currency crisis. If the infrastructure, for instance basic education, decentralisation, political stability and privatisation are not strong, then one may not blame a crisis triggering factor, say a particular hedge fund. This is because at that point there must be several candidate factors at least, which could do exactly the same job of finishing off the economy. Percolation can pull you down, percolation can also push you up. If all the basic and fundamental things are in place and intact, then one may fear neither malicious factors nor external influences. Progresses, when they come, will sometimes be seemingly abrupt. But they will all rest on the same secure and less obvious ground where all the basics lie. Karma is nothing but percolation. And the percolation, resting on potentials, is ever silent.

One may represent a country as a network. Then the network's structure is more important than its transient indicators.

I risk being thought of as blaming the victim in place of the culprit when I say that the Thai economy came down in 1997 not because of the attack on the Thai baht by George Soros, a Hungarian-American and owner of a powerful hedge fund in the US, but it came down because Thailand was structurally weak and had too many unnoticed or hidden problems.

Thus the Thai economy came down *when* Soros's hedge fund attacked us, but it came down *because* of our own structural weaknesses.

The same thing would have to happen anyway, around the same time as it did. According to the percolation theory, when a network is ripe, ready to undergo a change in phase, there are always several possible triggering events any one of which may do the job equally well.

In other words if it were not for the hedge fund, it would have been something else, *but*, and this is important, *the timing will not be far off*.

Economics is a network in which people are connected together forming clusters. These clusters are in the form of groups of people, companies, countries, and so on. Flowing from one person to another within this network is money. Conventional economics theories are money-centred which, despite the numerous indicators and indices they give us are not in accord with common reality. Both people and money are dynamic, but people are reasonably stable and fixed whereas money may in some degrees fluctuate. Moreover, the behaviour of money has its origin in the behaviour of people.

Plato in his *Phaedo* said that people usually worry themselves with things instead of with the power which drives them. In his argument he means the soul which drives the body. But the same is true with percolation. Percolation is the potential or power that drives things. People do not think about it but only look at indicators, which are nothing but effects of the potential. Different effects have different time-delays, so more likely than not heeding them without understanding brings a great amount of confusion.

All studies should lead us to a further understanding of things we study. Therefore astrology leads us to astronomy, which in turn leads to cosmology and then philosophy.

British by Chance

Kit Tyabandha

2/7/04

Ironically the British Explorer is only for a non-British.

3/7/04

I took 336 from Penzance to Birmingham, then 334 from Birmingham to Glasgow, and then 336 again from Glasgow.

4/7/04

At 2pm everyday the 534 coaches cross each other at New Castle, one going to Hull, the other one to Glasgow. There are numerous plays shown in Edinburgh. One of these is a *Jacob's bladder*. The New York Times talks about 'perpetually percolating imagination.' I had adopted the expression 'percolating' in philosophy and social science before. I wonder if they had the idea from my works. Another ad is for a comedy. It shows a picture of a man standing with his head covered in a bucket. The title says, 'Judge not.' Obviously the writer of the play has never read Rūmī. He would have otherwise found the answer to all his jests if he had..

Here you have to watch out for cows crossing the road. From Hull there is only 805 to Scarborough and 18-something to Beverly. We pass Bridlington, a seaside town with promenade and fun park. The ride on 524 is wonderful, passing through rain and sunshine. The countryside is superb.

From Scarborough there are only two coaches I may take. One of these is the 563 at 8:05am to London while the other the 322 at 8:20am to Brecon. I naturally want to take this second one which will bring me to Brecon at 19:10, not that I know firsthand where the town is, but because I have been to London many times already there is no need to go there again now. The coach driver wants me to stay at his mother-in-law's bed and breakfast, the Ivy Dene. I was not interested but did not say so. He was going to call her on his mobile phone but I told him that I would look around first. Anyhow, as the sign put up in the window says, there is no vacancy there. I decide to brave the cold night outside, waiting for the coach in the morning. I am convinced I had been here before with Aoife.

5/7/04

I don't know much about sufism but Rūmī is one of the best, whatever you call it.

Last night it must have been dark from 11pm until 3am only. There were two sources of light behind the cloud, one from the moon, then there was the sun.

The 322 goes from Scarborough via Birmingham and Cardiff to Brecon.

Brecon is an affable and lovable town. It makes you feel safe. There is a walking path called the Taff trail, which passes through the town. There is a Christ College founded in 1541 when the College of Prebends moved by order of Henry VIII from Abergwili near Carmarthen to Brecon.

The seat inside a small park was very cold during the night but not too much so.

6/7/04

A river passes through the town. The toilet here, near the promenade, opens 24 hours. Inside it is warm. I should have known last night. I could have been here all night where it is warmer than at the garden which, even though nicer under the stars, was dewy and cold.

At the Cathedral Church of St John Evangelist (Brecon Cathedral) tombstones are used as pavement stones. The is one 'IN memory of ANN Wife of ROGER EVANS of this Town Mason, who died Jan^{ry} 15^{th} 1821 Aged 34 Years.'

Dear Husband now my Life is past,
My love to you so long did last;
Now after me no Sorrows take,
But love my Children for my sake.

I took a 322 again from Brecon to Cardiff. It is a hot day today in Cardiff, a perfect day, clear sky with everywhere a hazy trail of jets flying.

Port Talbot is a small town. There are several petrol-chemistry and mineral processing plants just east of it. The town itself is near the sea

but not immediately seaside. From the coach station I could not find my way to the sea. The ground in this area is very knobbly. There are vast stretches of lumpy land.

At the station a man talked to me in Welsh, which I could not understand. I said that I was going to Newport. He must have repeated to me the name of that place in Welsh which I could not catch. I asked him the time. He asked a staff in Welsh, then told me in English it was two o'clock. I walked to the display screen to see what time it said. When I came back he was already gone, probably thought I tried to avoid his company. I shall learn to use Welsh.

My British Explorer expires today. But I could not get on the last coach before midnight to go to Manchester because the coach was full. So I got on another coach to go to, I think, Heathrow Airport. But a conductor got on board after we have started to tell me that I would need to buy a ticket anyhow from there to Manchester because it was my fault having missed my coach. So I grumbled and said what kind of hospitality this was, for the British Explorer was only sold to a non-British, and they did not let you reserve your seats but when you can not get on a coach because it was full it is your fault. I decided to get off at Golders Green. Rather walk the street and be with God than be among men and fear their heart.

I did not know how to go back to the Victoria Station, so I walked to the Golders Green Police Station to ask for the way. But it was closed. I picked up the phone in front of it and talked to a policeman. He told me to walk back towards Golders Green Station, then turn right at the first traffic light on to Hoop Lane, then right on to Golders Green Road, Brent St, right Church Road, two double-crossings right and Aprodrome Road or something. I lost count and asked him to repeat the information again. He told me again, but it was too complicated. The nearest police station that was open seemed to be very far away. The names of the streets are unfamiliar to me and I was not sure if I had got them right. A man and a woman passed by and said you never ask a police for help in this country.

7/7/04

I walked according to the vague instruction, and found my way all right in the beginning but lost it completely towards the end. There were suspicious cars which seemed to have passed me more than once. And there were others that stopped and waited for me to pass them by, which I tried to avoid.

Near a subway passing under a small road I lay down on the soft grass to get some sleep after it was light. Tonight it was not at all cold, which was fortunate for me because neither before or after this were there normally such nights as this. I managed to get very little sleep, then carried on walking along a motorway going north. I planned to hitch-hike my way to Manchester, but then changed my mind, bought a day tube ticket and went instead to the British Library.

I went back to Victoria Station and complained that the coach unkindly left me at Golders Green last night, and I had to walk all my way back here. I said that I missed my last coach but it was not my fault. The Custom Services of National Express then gave me a special ticket to go back to Manchester. The ticket had the company logo on top, then 'National Express Authority to Travel, ticket no. T1279823, Journey to Manchester, Reference WFGW, service number 540, Time of Travel 1700, Date of travel 07/07/04, number of pax. one adult only.'

But who will guard the guard?

Kit Tyabandha, Ph.D.

Everyone protects someone, be that someone be oneself or others. In this way we are all guards. But who will guard us?

Self-protection is an instinct. It is related to various other disciplines and involves many things, one of which is martial art. Every martial art is centred around instinct. And as a real instinct never intend to harm others, all martial arts are never about deliberate attacks.

Most *techniques* are based on *ways* and the essential part of these so-called *ways* have got much to do with the philosophy behind them. Each way can be described by two main attributes, namely *attitude* and *force*. The first one of these two deals with personal belief, the second one with Physics of the force.

Everyone is a guard. Some mainly guard themselves while others, for instance police, soldiers or security staffs, guard other people or society. For the latter, who will guard them?

Some of the more well-known martial arts are Aikido, Baguazhang, Bushidokan, Capoeira, Cha Yon Ryu, Cuong Nhu, Daito Ryu Aiki-Jujutsu, Hapkido, Hwa Rang Do, Iaido, Judo, Jujutsu, Kajukenbo, Kali (Escrima or Arnis), Karate, Kendo, Kenjutsu, Kenpo (American), Kempo (Kosho Ryu), Kempo (Ryukyu), Kobudo, Krav Maga, Kyudo, Lua, Moo Do, Muay Thai, Ninjutsu, Praying Mantis, ROSS, SAMBO, Sanshou, Savate, Shogerijutsu, Shuaijiao, Silat, Tae Kwon Do, Taijiquan, Wing Chun, Wushu (Gongfu), Xingyiquan and Yoseikan Budo.

Mọaïdaiï is not the same as *Thai boxing*. The former is older than the latter. *Boxing* means fighting with fists, while Mọaïdaiï makes extensive use of those parts of body other than fists. In fact it uses those other parts of body even more than it does fists, or *hands* as is a term often used in place of *fists*. Thais have rarely been specially renowned for heavy blows. Weightwise, none of us matchs western heavyweight fighters or Japanese sumos. Only agility has been our plus, and it stands out in Mọaïdaiï, together with all-aroundedness and straightforwardness. That last one gives Mọaïdaiï a charisma. A blow is a blow, clear and true even in tactics, sincere even when one surprises.

Though I try to put everything into the book, it can never replace a good teacher. One of the things he will do is asking you to swear not to misuse the Art, once you have learnt it. I do the same thing here. I ask you to swear before this book never to misuse what you shall have learnt from it. I assume that anyone who reads past this point is under this oath to your own consciousness. If you persevere in learning this art and then misuse it, or use it unhonourably, then — may what you did come home to roost.

History of Mọaïdaiï

Mọaïdaiï has been practised since the Sukhothai Era (1238 – 1377 AD.) Together with Daab Thai or the Thai art of weaponry, it had become a thing that every boy must learn when they reach puberty.

The Nine Weapons

The nine parts of the body can be used in 108 ways.

Head There are six usages.

Hand There are 24 movements.

When on guard the hands knead rather than clench. A fist can be kneaded differently for blows in different direction. When it is clenched it becomes suitable for a direct blow and unsuitable for a blow with the side of a fist. The force of a direct blow lands on two knuckles, namely that of the fore- and the middle-finger. Essentially when one kneads one's fist, one chooses which of the five knuckles to land the force on. The hands and the lower arms must be firm but relaxed. Force and control comes from those muscles in the upper arms.

For the very reason of a hand being kneaded instead of clenched to form a fist, the practice of using ropes to wrap up the hands is much more suitable than Mọaïdaiï than that of putting them in gloves. The rope provides both the necessary thickness and friction. It holds the hand together and gives every little bones support. One can tailor-made a tied fist to one's need. In the end result all fingers are left free, which is good when one wants to tune force in knuckles by kneading the hand. In short, one can do more with Mọaïdaiï with one's fists tied in ropes; one could do less with it putting them into gloves. In other words, a Mọaïdaiï fighter when with tied fists will be two times fiercer

than when he is with bare hands; with his hands in gloves he will be only half as fierce as when he is barehanded.

Elbows can be wielded in 30 different ways.

Knees No less than 12 actions exist.

Feet There are 36 combats using feet.

Tactics

There tactics for everything except the head. The following are tactics for hands, elbows, knees and feet. People in the past had obviously give the Art much thought, so the list seems complete. Whenever one tries to think up some new tactics one often finds that they are actually one of the old tactics in disguise. However, once in an advanced stage one should have a try in creating new tactics. Doing so is a creative thing which will enrich and uplift the Art and make it less mundane.
Tactics for hands

There are 15 tactics.
Tactics for elbows
Tactics for knees
Tactics for feet

Roots

To Canada

There are 15 roots.

Zigzag far-range protect. Step to the outward side of a punch, grab the wrist and the upper arm.

Entering eagle close-range protect. Get close, ward punch at arm position with upper arm on the same side, the other hand attacks.

Java's spear far-range elbow. Side-step to outward side, attack with the closest elbow.

Krish piece close-range elbow.Get close, ward arm with arm, the other elbow at chest or ribs.

Mountain lifting Step the foot on the opposite side of a punch, with the whole body leaning forward and arm underside up put the fist on the same side of the punch under chin.

Old man support Body leaning forward, hand or fist on the other side of the punch at chin.

Mon's pole Tip or heel of a foot at abdominal area to ward off a punch.

Crochet Step the foot on the opposite side of the coming leg, elbow at the front bone of the leg.

Crocodile's tail When the momentum leads back frontwards, follow it and attack high with the back of a leg or heel.

Snapped trunk Coming leg against upper arm, lock with the arm, the other elbow both protect the head and attack upper leg or the knee bone, then face.

Naga's tail Grab the ankle and lift up, knee at the underside muscles of the lower leg.

Somersault sparrow Ward a kick at the thigh with surging foot.

Extinguish Get outwards of a punch, attack inwards with one arm or punch at eye or face.

Troll and ape Inwardly ward a punch, step back and answer a leg with elbow at thigh, arms to receive the coming down elbows, off-balance.

*Neck lock*Step close, grab the back of the neck and pull down, knee upwards to meet.

Derivatives

There are 15 derivatives, namely

Elephant tusk Step outward of a punch, one arm gets inwards, with under-arm upwards fist at chin and elbow to chest.

Footing face Straight kick at chin, then foot the face coming in with momentum.

Eloping giant Get close, grab the waist and pivoting at the hip up-turn the other party.

Rama's bow Arm against coming in elbows, fist at chin with arm turning upwards.

Brook crossing Side-step a high leg, get towards the backside, surge foot at knee or thigh.

Inviting deer When missed with a kick, turn around and surge a high other foot to meet possible attack.

Rolling earth Meet a leg with an arm, turn around following the momentum and put elbow at face or chin.

Tunnelling Naga Dodge under a high leg, surge a foot at backside of the knee.

*Hanuman's ring*One arm protects or clears the guards, the hand or fist of the other arm at chin.

Vietnamese nets Dodge or let a leg pass or grab its ankle, leg at the backside of the knee of the other leg on the ground.

*Propped pole*Escape a high leg, a foot surges at just above the knee of the leg standing on the ground.

*Wingless swan*Being inwards of an arm, bring the opposite elbow down on the upper arm.

Tattooing lei Remaining inside of an arm, the elbow on the opposite side at chest and repeat.

Sweeping priest Get towards the backside, sweep the back legs with one leg.

Slicing gourde Remaining inside of an arm, the other elbow at face, followed by another one at face and repeat.

Apart from these there are

Mondho on throne While an upper leg remains in a lifted position, follow it up like sitting on it, elbow down to top of head.

Rama snap Grab a wrist, the other hand grab a bended elbow, twist the elbow and arm to front.

Rama mantra Jump and surge a foot at face.

Diamond cut Grab an ankle, put the leg on the same side on that leg and put weight on it.

Windmill Get outwards of a knee, one arm gets under the knee and lift up while the other arm push the body backward.

Geometric Algorithm

Kit Tyabandha, Ph.D.

The altitude lines of a triangle are concurrent. The bisectors of the angles of a triangle are concurrent. Ceva's theorem says that, all the three lines in a triangle which contain a vertex and a point on its opposite side are concurrent if and only if no two among them are parallel and the product of the three ratios of division of the sides made in one direction around the circumference of the triangle is one. In Gergonne's theorem, the three lines of a triangle which are made by the vertices and the points of tangency of the incircle on the side opposite to them are concurrent. The intersection between the three lines tangent to the circumcircle of a triangle and the sidelines opposite to them are collinear.

A point is an extreme point of a plane convex set s unless it lies in a triangle which has vertices in s but is no vertex of the triangle. A ray from inside a bounded convex figure intersects the boundary of the latter at exactly one point. Consecutive vertices of a convex polygon exist in sorted angular order about any interior point. A subfacet of a simple polytope is shared by two and only two facets. Two facets share a subfacet if and only if the latter is determined by $d-1$ vertices in their set; these two facets and the subfacet are called adjacent. A line segment defined by two points is an edge of the convex hull if and only if all other points of the set lie on, or to one side of it.

The diameter of a convex figure is the largest distance between parallel lines of support. The diameter of its convex hull determines the diameter of a set. Every vertex of the Voronoi graph is the intersection of three of its edges. Every nearest neighbour of a Voronoi polygon defines an edge.

VT and the triangulation of its nuclei are dual to each other. A Voronoi graph on n points has at most $2n-5$ vertices and $3n-6$ edges. The convex hull of a Voronoi graph on n can be found in linear time.

Modern programming philosophy puts much emphasise on modularity of a programme and on information hiding of modules. Though undoubtedly information hiding can be good for the finished products, during the course of development it sometimes works against yourself

when you try to pinpoint an error in order to debug. Some modularisations are more about hierarchies than simplicity. Whenever this is the case, it is necessary to unconventionally seek a simpler path.

The explicit equation in 2-d is $y = mx + c$ or $ax + by + c = 0$. Imposing the constraint $a^2 + b^2 = 1$, that is multiplying all the terms by $(a^2 + b^2)^{-1/2}$, puts the equation into the canonical or normalised form. This makes $a = \cos\alpha$, $b = \cos\beta$ and $c = -r$, where a and b are directional cosines, *i.e.* the cosines of the angles that the normal line makes with the x and y axes respectively. Examples of possible conventions are to have a normal line point towards outside of the region, to have the line direction always to the right of the normal vector, or to keep c positive always.

A parametric form of line equation in 2-d is by introducing a third variable t and write the equations as $x = x_0 + ft$ and $y = y_0 + gt$, where (x_0, y_0) is the point on the line corresponding to $t = 0$. A line through a point p which makes angles α and β with the x and the y axes respectively has the parametric equations $x = x_p + t\cos\alpha$ and $y = y_p t\cos\beta$. One convention is to vary t from 0 to 1 over a line segment, another is to normalise it by multiplying its coefficient by $(f^2 + g^2)^{-1/2}$.

An implicit line equation $ax + by + c = 0$ can be turned into a parametric form as $x = -ac/(a^2+b^2)^{1/2} + bt$ and $y = -bc/(a^2+b^2)^{1/2} - at$. And a parametric line described by $x = x_0 + ft$ and $y = y_0 + gt$ is converted into the implicit form as $-gx + fy + (x_0 g - y_0 f) = 0$

The implicit plane equation in three dimensions is $ax + by + cz + d = 0$. The parameters can be found by using Cramer's rule, $a = \det(1, y_i, z_i)$, $b = \det(x_i, 1, z_i)$, $c = \det(x_i, y_i, 1)$ and $d = \det(x_i, y_i, z_i)$, which gives $a = y_1 z_{32} + y_2 z_{13} + y_3 z_{21}$, $b = z_1 x_{32} + z_2 x_{13} + z_3 x_{21}$, $c = x_1 y_{32} + x_2 y_{13} + x_3 y_{21}$ and $d = x_1(y_2 z_3 - y_3 z_2) + x_2(y_3 z_1 - y_1 z_3) + x_3(y_1 z_2 - y_2 z_1)$.

A normalised form has a constraint $a^2 + b^2 + c^2 = 1$. This amounts to multiplying its implicit equation by $(a^2 + b^2 + c^2)^{-1/2}$ to get $\alpha x + \beta y + \gamma z + \delta = 0$. Here α, β and γ are cosines of the angles which the normal to the plane makes with the coordinate axes. The distance between two parallel normalised planes is $\delta_2 - \delta_1$. A normalised implicit plane equation can be used to represent a planar half-space by multiplying every tems by -1 and then assign a convention that the vector formed by the direction cosines always points towards the outside or the inside of the region.

The distance from a point to a plane, if the plane is $ax+by+cz+d=0$ and the point is (x_p, y_p, z_p), is

$$r = \left[\frac{(ax_p + by_p + cz_p + d)^2}{(a^2 + b^2 + c^2)}\right]^{1/2}$$

The intersection of two planes, from the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, is $x = x_0 + ft$, $y = y_0 + gt$ and $z = z_0 + ht$ where $f = \det(b_i, c_i)$, $g = \det(c_i, a_i)$ and $h = \det(a_i, b_i)$, $i = 1$ and 2.

The intersection of three planes is found by Algorithm 1, Here the minor matrices a^{ij} is δ^{ab}_{23}, δ^{ac}_{23}, or δ^{bc}_{23} as the case may be.

Algorithm 1 *Intersection among three planes.*

$\Delta \leftarrow \sum_{(a,b,c)}(-1)^{i+j} a_{ij} a^{ij}$;

if $|\Delta| < \epsilon$ **then**

at least two of the planes are parallel;

else

$x \leftarrow (b_1\delta^{dc}_{23} - d_1\delta^{bc}_{23} - c_1\delta^{db}_{23})/\Delta$;

$y \leftarrow (d_1\delta^{ac}_{23} - a_1\delta^{dc}_{23} - c_1\delta^{ad}_{23})/\Delta$;

$z \leftarrow (b_1\delta^{ad}_{23} + a_1\delta^{db}_{23} - d_1\delta^{ab}_{23})/\Delta$;

endif □

The intersection between a line and the plane $ax + by + cz + d = 0$ is $(x_1 + x_{12}a, y_1 + y_{12}a, z_1 + z_{12}a)$, where $a = -(ax_1 + by_1 + cz_1 + d)/(ax_{12} + by_{12} + cz_{12})$, $x_{12} = x_2 - x_1$ and similarly for y_{12} and z_{12}.

The area of a circle is πr^2 and that of its segment is $\theta r^2/2$. A segment is its pie cut reaching its centre while a sector is a plane slice through the sphere. The area of a sector is this area subtracted by that of a triangle, or $r^2(\theta - \sin\theta)/2$. The centre of gravity or the centroid lies on the bisector of the central angle with the distance of $4r\sin(\theta/2)/3\theta$ for a sector and $4r\sin^3(\theta/2)/3(\theta - \sin\theta)$ for a segment.

The volume of a pyramid is $Ah/3$, where A is the area of base and h is the height of the pyramid. The volume of a sphere is $4\pi r^3/3$, and the distance from its centroid to the sphere centre is

$$(3r/4)\left[\frac{\sin^4(\theta/2)}{(2-3\cos(\theta/2)+\cos^3(\theta/2))}\right]$$

That of a sector of a sphere is $(\pi r^3/3)(2-3\cos(\theta/2)+\cos^3(\theta/2))$ The volume of a tetrahedron is $V=(1/6)\det(x_{12},x_{13},x_{14};y_{12},y_{13},y_{14};z_{12},z_{13},z_{14})$, where $x_{ij}=x_j-x_i$,or $V=(1/6)\det(x_i,y_i,z_i,1)$. The former is limited to the case of three dimensions, and is in fact $V=(1/6)(a\times b)\cdot c$, where a, b and c are respectively the lines from O to A, B and C in a tetrahedron OABC.

Generalising the latter to higher dimensions, we have the volume of a d-dimensional simplex $V=(1/d!)\det(x_{ij},1)$, where x_{ij} is now $(x_j)_i$, $1\le i\le(d+1)$ and $1\le j\le d$. Here the information found in existing literature seems to be wrong, some lists the multiplying factor as $1/d$, some simply uses $1/6$ throughout all $(d\ge 3)$! Tiyapan (2004) mentions in more detail how I arrived at the value used here.

Some of of the algorithms found in literature are the following. The algorithm to find whether a point is inside a polygon, Algorithm 2. The arbitrary line l here, which is taken for simplicity to be horizontal, passes through z.

Algorithm 2 *Point inside a polygon.*

$r \leftarrow 0$;

for $i=1$ to n **do**

if edge i and l not parallel **then**

if i intersects l to the left of z at any point except its lower extreme **then**

$r \leftarrow r+1$;

endif

endif

if r odd **then**

z is internal to p;

else

z is external;

endif

endfor □

To find the inclusion in a convex polygon, $q \in p$ being a known fixed point within the polygon, find the wedge in which z lies by doing a binary search and test whether $\angle(zqp_{i+1})$ is a right turn- while $\angle(zqp_i)$ a left turn angle. If $\angle(p_ip_{i+1}z)$ is a left turn angle, then z is inside p.

The Euclidean minimum spanning tree may be obtained by Algorithm 3.

Algorithm 3 *Euclidean minimum spanning tree.*

$f \leftarrow 0$;

for i from 1 to n **do**

$s(p_i) \leftarrow 0$;

$f \leftarrow p_i$;

endfor

while f contains more than one number **do**

$t \leftarrow f$;

if (s(t)=j) **then**

clean up;

$j \leftarrow j + 1$;

endif

$(u, v) \leftarrow$ shortest unselected edge incident on t, $u \in t$;

$t' \leftarrow$ tree in f containing v;

$t'' \leftarrow$ **merge** $(t, t')]$;

delete (t') from f;

$s(t'') \leftarrow \min(s(t), s(t')) + 1$;

$f \leftarrow t''$

endwhile ▫

To rotate $v = (v_1, v_2)^{\mathrm{T}}$ to $v' = (v_1', v_2')^{\mathrm{T}}$, use $v^{\mathrm{T}} = Av$ where $A = [\cos\theta, -\sin\theta; \sin\theta, \cos\theta]$ is the transformation matrix and θ is the anti-clockwise angle of the rotation.

To rotate a general line in three dimensions by θ around an arbitrary axis, the transformation matrix becomes $A = T^{-1}RT$, where

$$\begin{aligned} R(\theta) &= R_x^{-1}(\alpha) R_y^{-1}(\beta) R_z(\theta) R_y(\beta) R_x(\alpha) T \\ &= R_x(-\alpha) R_y(-\beta) R_z(\theta) R_y(\beta) R_x(\alpha) \end{aligned}$$

The translation T translates one point of the line to the origin and R_x rotates around the x-axis by α which puts u on to the xz axis, R_y around y-axis by β and puts u' on to the z axis, and R_z around z-axis. Since the vector v is (x_{12}, y_{12}, z_{12}), we have the a, b, c and the unit vector $u = (a, b, c)/|v| = (x_{12}, y_{12}, z_{12})/|v|$. Then it follows that $\cos\alpha = c/d$, $\sin\alpha = b/d$, $\cos\beta = d$ and $\sin\beta = -a$. All of these can perhaps be summarised as a linear procedure in Algorithm 4.

Algorithm 4 *Rotation in three dimensions*

$v \leftarrow (x_{12}, y_{12}, z_{12})$;

$m \leftarrow |v|$; $a \leftarrow x_{12}/m$;

$b \leftarrow y_{12}/m;$

$c \leftarrow z_{12}/m;$

$d \leftarrow (b^2 + c^2)^{1/2};$

$r_{xx} \leftarrow c/d;$

$r_{xy} \leftarrow b/d;$

$r_{yx} \leftarrow d;$

$r_{yy} \leftarrow -a;$

$T \leftarrow [\Im(3), -(x_1, y_1, z_1)^{\mathrm{T}}; 0(3)^{\mathrm{T}}, 1];$

$R_x \leftarrow [1, 0(3)^{\mathrm{T}}; 0, r_{xx}, -r_{xy}, 0; 0, r_{xy}, r_{xx}, 0; 0(3)^{\mathrm{T}}, 1];$

$R_y \leftarrow [r_{yx}, 0, -r_{yy}, 0; 0, 1, 0, 0; -r_{yy}, 0, r_{yx}, 0; 0(3)^{\mathrm{T}}, 1];$

$R_z \leftarrow [[\cos\theta, -\sin\theta; \sin\theta, \cos\theta], 0(2,2); 0(2,2), \Im(2)];$

$R \leftarrow T^{-1} R_y^{-1} R_z R_y R_x;\ v' = Rv.$ □

Also in three dimensions, the rotation around the x-axis is

$$R_x = [\{r_{11} = 1, m_{23} = [\cos\theta, -\sin\theta; \sin\theta, \cos\theta]\}]\,,$$

around y-axis is

$$R_y = [\{r_{22} = 1, m_{13} = [\cos\theta, \sin\theta; -\sin\theta, \cos\theta]\}]$$

and around z-axis is

$$R_z = [\{r_{33} = 1, m_{12} = [\cos\theta, -\sin\theta; \sin\theta, \cos\theta]\}]\,.$$

The minor containing parts of the i^{th} and j^{th} rows and columns is m_{ij}.

The right hand coordinate system is where a 90° rotation around the x-, y- and z-axis bring respectively the y- to z-, z- to x- and x- to y-axis. Scaling and translating a vector v in three dimensions amounts to

calculating $[v'; w] = A[v; w]$, where A is respectively $[\Im(3)s, 0(3); 0(3)^{\mathrm{T}}, 1]$ and $[\Im(3), \Delta v; 0(3)^{\mathrm{T}}, 1]$, $\Delta v = (\Delta x, \Delta y, \Delta z)^{\mathrm{T}}$.

A quaternion can be described as a pair (s, v) of a scalar s and a vector $v = (a, b, c)$. The rotation by θ around an axis in the direction of a unit vector u is then the quaternion $(\cos\theta/2, u\sin\theta/2)$. Let $q = (s, v)$. Then $q^{-1} = (s, -v)$. The multiplication of quaternions is $q_1q_2 = (s_1, v_1) \cdot (s_2, v_2) = (s_1s_2 - v_1 \cdot v_2, s_1v_2 + s_2v_1 + v_1 \times v_2)$, where the cross product is described in minors as $v_1 \times v_2 = [\delta^{yz}; -\delta^{xz}; \delta^{xy}]$.

Let q represent a rotation. Then a vector p is rotated to p' by $P' = qPq^{-1}$, where P and P' are respectively $(0, p)$ and $(0, p')$. In simplified words, this means $p' = s^2p + (p \cdot v)v + 2s(v \times p) + v \times (v \times p)$. Then we have the transformation matrix for the general rotation around u in three dimensions,

$$R_u(\theta) = \begin{bmatrix} (1 - 2b^2 - 2c^2) & (2ab - 2sc) & (2ac + 2sb) \\ (2ab + 2sc) & (1 - 2a^2 - 2c^2) & (2bc - 2sa) \\ (2ac - 2sb) & (2bc + 2sa) & (1 - 2a^2 - 2b^2) \end{bmatrix}$$

where $s = \cos\theta/2$ and $v = (a, b, c) = u\sin\theta/2$. Furthermore, if q_1 is a rotation by θ_1 around v_1 and likewise q_2 by θ_2 around v_2, then $q_3 = q_2q_1$ is a rotation by $\theta_3 = 2\cos^{-1}s_3$ around v_3 such that $\sin\theta_3 \geq 0$.

Quaternion is an extension of complex number to higher dimensions where there are three imaginary parts instead of one. It is defined as $q = s + ia + jb + kc$, where a, b, c and s are real numbers, $i^2 = j^2 = k^2 = -1$ and $ij = -ji = k$.

Let a plane be described by $(v - p) \cdot n = 0$, where p is a point on-, and n a perpendicular to the plane. This means that, for all points v lying in the plane (p, n), $(v - p) \cdot n = 0$. If the plane (p, n) is transformed into (Ap, m), then Av lies in (Ap, m) and consequently $A^{\mathrm{T}}m = n$ or $m = (A^{\mathrm{T}})^{-1}n$.

A vector normal to the surface remains normal if it is transformed by $(A^{\mathrm{T}})^{-1}$. When a transformation matrix A has the property $A = (A^{\mathrm{T}})^{-1}$, for example rotation, all surface normals remain normal. But in general this equality does not hold, so we have for instance the non-uniform scaling where these normals cease to be normal.

To test for the intersection between a ray and a triangle using Plücker's coordinates is described in Algorithm 5.

Algorithm 5 *Ray and triangle intersection.*

find Plücker's coordinates for vertices and the ray;

test ray against each of the edges;

if ray hits an edge, passes all of them clockwise or all of them counter-clockwise **then**

ray intersects the triangle;

else

ray and triangle intersect not;

endif □

If these vertices are v_1, v_2 and v_3, and the ray is $r = r_{12} = r_2 - r_1$, where r_1 and r_2 are any two points on the ray, then Plücker's coordinates for the vector v_{12} are $(u, v) = (v_2 - v_1, v_2 \times v_1)$, and similarly for v_{23}, v_{31} and r_{12}. For the test between the ray and each edge, find $c = u_r \cdot v_i + v_r \cdot u_i$. Then the ray r counter-clockwise passes the edge i, hits it or clockwise passes it respectively as $c < 0$, $c = 0$ or $c > 0$.

A 3-d line can be represented by the six numbers that come with coordinates of two distinct points, or by the eight numbers that come with the coordinates of two distinct planes. Plücker's coordinates, however, provides a mean which suits geometrical computation better than both of these. It redefines the coordinates as $u = p - q$ and $v = p \times q$, where p and q are two points on a line, neither quantity of which depends on p or q.

Let a tetrahedron has its vertices at a_i, $i = 1$ to 4. Then the centre of its circumsphere is at $a_1 + \delta$ and its corresponding radius $r = |\delta|$, where

$$\delta = \left[|a_{12}|^2(a_{13} \times a_{14}) + |a_{13}|^2(a_{14} \times a_{12}) + |a_{14}|^2(a_{12} \times a_{13})\right] / 2|a_{12}^{\mathrm{T}}; a_{13}^{\mathrm{T}}; a_{14}^{\mathrm{T}}|.$$

A triangle $\triangle p_1p_2p_3$, where $p_i = (x_i, y_i)$ and $i = 1$ to 3, has an area $A = |x, y, 1(3)|$, which is positive if and only if $\triangle p_1p_2p_3$ forms a counter clockwise cycle or $\angle p_1p_2p_3$ is left-turned. Or equivalently the area is $A = \det(x_{12}, x_{13}; y_{12}, y_{13})$, which is positive if the points are in anti-clockwise order of the indices and negative otherwise. Or the area

is $A = [s(s-d_1)(s-d_2)(s-d_3)]^{1/2}$, where the the semi perimeter s is $s = \sum_i d_i/2$, where d_i, $i = 1$ to 3, are the lengths of the three sides of the triangle.

For a convex polygon, the area can be found by adding together the n triangles formed by any two adjacent vertices and one fixed point within the polygon. Here n is the number of vertices it contains. Or it can be found by adding the $(n-2)$ triangles formed by one fixed vertex and any two adjacent vertices of those remaining. Another way of finding the area is $A = (1/2)\sum_{i=0}^{n-1}(x_i y_{i+1} - y_i x_{i+1})$, where (x_i, y_i) are vertices. Rearranging to make it faster and more accurate, $A = (1/2)\sum_{i=0}^{n-1}((x_i + x_{i+1})(y_{i+1} - y_i))$. If the dimension of the polygon is higher than two, $A = (1/2)\left|N \cdot \sum_{i=0}^{n-1}(v_i \times v_{i+1})\right|$, where N is a unit vector normal to the plane. The area of a polygon can also be computed, without loosing generality, by subtracting the area under its lower edges by that of its upper edges.

A vector normal to a plane is simply $n = v_{12} \times v_{13}$.

The ordering of vertices on a face of a polygon is done for the purpose of drawing it or for finding vertice pairs which form edges. This can be done in two ways. One is to get inside the polygon and look at all the vertices around comparing their angles relative to one another. Another one is to look at the polygon from a distance and compare their angles as before, as well as their distance from the viewing point. The angle is $\theta = \arccos[(v_1 \cdot v_2)/(|v_1||v_2|)]$, where v_1 and v_2 are vectors to vertices from the distant point, and θ the angle between them.

To find the convex hull in three dimensions, one may use Algorithm 6.

Algorithm 6 *Convex hull in three dimensions.*

sort s by x_1 such that $x_i(p_i) < x_i(p_j)$ if and only if $i < j$;

if $|s \leq k|$ **then**

  construct $c(s)$;

else

  $s_1 \leftarrow \{p_1, \ldots, p_{\lfloor n/2 \rfloor}\}$;

$s_2 \leftarrow \{p_{\lfloor n/2 \rfloor}, \ldots, p_n\}$;

$p_1 \leftarrow c(s_1)$;

$p_2 \leftarrow c(s_2)$;

$p \leftarrow$ **merge** p_1 and p_2;

endif □

Here $c(s)$ is the convex hull of s. Two convex hulls are merged with each other by first constructing a cylindrical triangulation T which supports p_1 and p_2 along two circuits e_1 and e_2 respectively, then remove from both p_1 and p_2 the portions which have been obscured by T.

The following Jarvis's march algorithm, Algorithm 7, finds a convex hull in two dimensions.

Algorithm 7 *Convex hull in two dimensions*

$p_1 \leftarrow$ the lowest point in s;

$q_1 \leftarrow$ the highest point in s;

while next point $\neq q_1$ **do**

find $p_i \in s$, $i = 2, 3, \ldots$, with an increasing order of the polar angles

with respect to p_1;

endwhile

while next point $\neq p_1$ **do**

find $q_i \in s$, $i = 2, 3, \ldots$, with an increasing order of the polar angles

with respect to q_1 and the negative x axis;

endwhile □

The polar angle is an angle with respect to the positive x-axis. The lowest and the highest points are on the convex hull.

Algorithm 8 is the quick hull algorithm.

Algorithm 8 *Quick hull algorithm.*

$l \leftarrow (x_0, y_0)$;

$r \leftarrow (x_0, y_0 + \epsilon)$;

if $s = \{l, r\}$ **then**

return (l, r);

else

find $k \in s$ that gives $\max A_{\triangle klr}$ or $(\max A_{\triangle klr}$ and $\max \angle klr)$;

$s^1 \leftarrow p \in s$, such that p is on the left of $\overrightarrow{lh}$;

$s^2 \leftarrow q \in s$, such that q is on the left of $\overrightarrow{hr}$;

$\{h\} \leftarrow (s^1; l, h)$;

$\{h\} \leftarrow (s^2; h, r) - h$;

endif □

The convex hull is $\{h\}$. The points l and r are with respectively the smallest and the largest abscissa. In other words they are the left-most and the right-most points. And k is the furthest point with respect to l and r.

Let $p = \{p_1, p_2, \ldots, p_n\}$ be a set of n generators in ν-dimensional space, the coordinates of which are (x_{ij}), $i = 1$ to d and $j = 1$ to n. Then

the Delaunay tessellation in d dimensions which spans p is generated by,

for $j = 1$ to n **do**

$\{q\} \leftarrow q_j = (x_{ij}, \sum_j x_{ij}^2)$;

endfor

$h \leftarrow c(q)$;

project all the lower n-faces of $c(q)$ parallel to the d^{th} axis on to the original n-d space;

□

Okabe *et al* (1992) give a good review of algorithms for generating VT's. Given the set of generator points $\{p_i\}$, $i = 1$ to n, a brute force albeit simple method generates for all i and j from 1 to n the $(n-1)$ half planes $h(p_i, p_j)$, $1 \leq j \leq n$, $i \neq j$, and then proceeds to construct all $\mathcal{V}(p_i)$ of the VT from their common intersections.

On the other hand, the following Algorithm 9 is the quaternary incremental method whose inputs comprise the $(n-3)$ generators p_i, $i = 4$ to n, where all the p_i are in $s = \{(x, y) | 0 \leq x, y \leq 1\}$, and three additional generators $p_1 = (0.5, 0.5(1 + 3\sqrt{2}))$, $p_3 = (0.25(2 - 3\sqrt{6}), 0.25(2 - 3\sqrt{2}))$ and $p_3 = (0.25(2 + 3\sqrt{6}), 0.25(2 - 3\sqrt{2}))$.

Algorithm 9 *Quaternary incremental method.*

$k \leftarrow \min(k_i)$ such that $k_i \in \Im^+$ and $n < 4^{k_i}$;

$s_{ij} \leftarrow (i - 1, i/2^k, j - 1, j/2^k)$;

construct a quaternary tree;

scan leave buckets from left to right, top to bottom, **put** generators in buckets;

$\{p_i\}$ **do** quaternary reordering on the generators;

$\mathcal{V} \leftarrow$ **construct** Voronoi for p_1, p_2 and p_3**;**

for $i = \mathbf{4}$ **to** n **do**

repeat $\leftarrow 1$**;**

while repeat **do**

find p_α such that $\mathrm{d}(p_\alpha, p_\ell) = \min_j \mathrm{d}(p_j, p_\ell)$**;**

if $\mathrm{d}(p_m, p_\ell) < \mathrm{d}(p_\alpha, p_\ell)$ **then**

repeat $\leftarrow \neg$**repeat;**

elseif $p_m \leftarrow p_\alpha$ **do** nothing**;**

else

repeat $\leftarrow \neg$**repeat;**

endif

endwhile

$\{w_{i1}, w_{i2}\} \leftarrow$ the intersections between the perpendicular

bisector of $p_i p_l$ and $\Omega(\mathcal{V}(p_i))$, $1 \leq i \leq \ell - 1$**;**

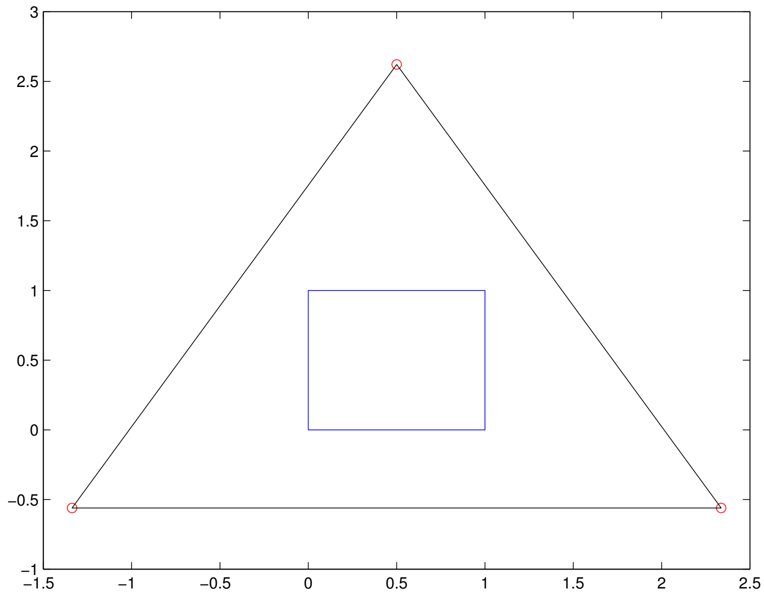
$\Omega \leftarrow$ **construct** the boundary of $\mathcal{V}(p_i)$ formed by these $\overline{w_{i1} w_{i2}}$**;**

$\{\mathcal{V}_\ell\} \leftarrow (\{\mathcal{V}_{\ell-1}\} \bigcup \Omega) - \{\mathcal{V} | \mathcal{V} \in \{\mathcal{V}_{\ell-1}\}, \mathcal{V}$ is within $\Omega\}$**;**

endfor

$\{\mathcal{V}\} \leftarrow \{\mathcal{V}_n\}$**;**

$\square$



The construction of s_{ij} is such that $s_{ij} = (i-1, i/2^k] \times (j-1, j/2^k]$, where $i, j \in \Im^+$, i and j are 1, 2, … up to 2^k. The nearest neighbour search finds from ℓ generators, $p_1, \ldots, p_\ell$, and $\{\mathcal{V}_{\ell-1}\}$, the generator point p_m such that $\mathrm{d}(p_m, p_\ell) < \mathrm{d}(p_m, p_i)$ for all $i = 1$ to ℓ, $i \neq m$ and $i \neq \ell$; p_j are all generators adjacent to $\mathcal{V}(p_i)$. The boundary growing procedure gives the sequence of boundaries $\{\Omega\}$. The additional generators mentioned in the procedure is graphically shown in Figure 1.

If we draw a horizontal line through each point in two dimensions, or a horizontal plane each point in three dimensions, we divide the space into slabs the line segments within which do not intersect one another. In three dimensions, and for a polygonal model of porous media, we can for instance divide the model into such slabs, then find the effective cross sectional area of each slab, and then determine where the bottle neck to the flow occurs within the media. Whether this would produce the correct determination of the pressure of flow across the material is another matter because flows through porous media may be governed by the combination of the various tortuous paths through the pores, the interrelationship of which can be complicated.

These slabs provide another method in finding the area, or in three dimensions the volume, of each cell of the tessellation. Here the cell is divided into slabs, and then the area or volume of each section calcu-

lated.

In three dimensions, rotation around the x-axis is done by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

around y-axis by

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

and around z-axis by

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The following programme finds the area and plane parameters of a face from a matrix containing the list of the coordinates of the ordered vertices. Its synopsis is `(a,p)=findfarea(v)`, where `a` is the face area, `p` the list of the plane parameters and `v` the coordinate matrix.

```
% findfarea.m, finds face area, (c) Kit Tiyapan, February, 2001.
function [fca, ppr] = findfarea(odvc)
nvthf=size(odvc,1);
stp=floor(nvthf/3);
ndpt =1+stp;
rdpt =1+2*stp;
nmvc= cross((odvc(ndpt,:)-odvc(1,:)),(odvc(rdpt,:)-odvc(1,:)));
clov =[odvc; odvc(1,:)];
xsq =nmvc(1,1)*nmvc(1,1);
ysq =nmvc(1,2)*nmvc(1,2);
zsq =nmvc(1,3)*nmvc(1,3);
znmvc =sqrt(xsq+ysq+zsq);
nmnmv =nmvc/znmvc; socp =0;
for i=1:nvthf,
  socp =socp+cross(clov(i,:),clov((i+1),:));
end
fca =(abs(dot(nmnmv, socp)))/2;
nvct =ones(3,1);
crdm =[odvc(1,:);
odvc(ndpt,:);
odvc(rdpt,:)];
aprm =det([nvct, crdm(:,2), crdm(:,3)]);
bprm =det([crdm(:,1),  nvct, crdm(:,3)]);
cprm =det([crdm(:,1), crdm(:,2), nvct]);
dprm =det([crdm(:,1), crdm(:,2), crdm(:,3)]);
ppr =[aprm, bprm, cprm, dprm];
```

Real problems are algebraical or analytical whereas computer simulations are arithmetical and numerical (*cf* Knuth, 1997). In between these two lie computer programmes. Therefore the latter are mapping from the analytical to the arithmetical world. There are different ways to solve a problem analytically, and there are different ways numerically. Therefore numerical study by simulation is an endless pursuit. Several programmes given by Tiyapan (2004) do the same job, but each one does it differently.

Hamiltonian's are essentially a function that maps N bodies pairwise to account for all pairs of their mutual interactions. They occur in most fields where there are particles interacting with each other. In quantum mechanics, for example, the time independent Schrödinger equation for a system of N particles interacting via the Coulomb inter-

action is $H\Psi = E\Psi$, where the Hamiltonian is

$$H = \sum_{i=1}^{N}\left(-\frac{\hbar}{2m_i}\nabla_i^2\right) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}^{N}\frac{z_i z_j}{4\pi\epsilon_0|r_i - r_j|}, \tag{1}$$

where Ψ is the N-body wavefunction, z the charges of the individual particles and E the energy of either the ground or an excited state of the system. Similarly from Rushbrooke and Morgan (1961), using their notations, the Hamiltonian of the Ising problem is

$$\mathcal{H} = -2J\sum_{\langle i,j\rangle} s_3^{(i)} s_3^{(j)} - g\beta H \sum_{(i)} s_3^{(i)}, \tag{2}$$

where β is the Bohr magneton, g the gyromagnetic ratio and J the magnitude of the exchange interaction.

When simulating such systems, the number of pairwise summation terms can be reduced by half because they represents a symmetric matrix. The total number of terms is thus reduced from $N(N-1)$ to $N(N-1)/2$ (*cf* Wray *et al*, 1983). Because $i \neq j$, all the diagonal components of the matrix are excluded, which makes the number of pairs $n^2 - n = n(n-1)$. In a programme, this is equivalent to two if statements, one embedded within the other in the form $i(j(\cdot))$, where the index i runs from 1 to $(n-1)$ and j from $(i+1)$ to n; this has been discovered from experience, as can be seen by comparing the present work with the earlier one (Tiyapan, 1995, ℵ).

Convex Hull

The set of extreme points E in some superset S is the smallest subset of S such that the convex hulls of both E and S are identical. Extreme points never lie in a triangle.

The Graham's scan algorithm (*cf* Preparata and Shamos, 1985) positions itself in the midst of the points and then scans around in one direction. It determines three points in turn, and rejects a mid point among the three if the angle made there is reflexive, *i.e.* α such that $\alpha \geq \pi$ in the anti-clockwise direction. In other words, an angle is a right turn if it is reflexive; it is a left turn otherwise. The algorithm is shown in Algorithm 10. Here $\{h\}$ is a stack which contains the points on convex hull

Algorithm 10 *Graham's scan.*

$i \leftarrow 1$;

$j \leftarrow 2$;

$k \leftarrow 3$;

while there exist unprocessed points **do**

if $\angle p_i p_j p_k \geq \pi$ **then**

$j \leftarrow i$;

$i \leftarrow i - 1$;

else

$i \leftarrow j$;

$j \leftarrow k$;

$k \leftarrow k + 1$;

endif

endwhile □

The algorithm starts from a known extreme point. It can also start from two points which are known to be extreme, in which case the space is divide into an upper- and a lower hulls. Similarly to the Graham's scan, Jarvis's march finds one extreme point after another as it wraps a line around the convex hull. The Quickhull algorithm in two dimensions is described in Algorithm 11.

Figure 11 *Quickhull in two dimensions.*

$l \leftarrow$ the point with the smallest abscissa;

$r \leftarrow$ the point with the largest abscissa;

$\{s\} \leftarrow$ all points above $\overline{lr}$;

$\{s\} \leftarrow$ all points below $\overline{lr}$;

while there remains an unprocessed s_i in $\{s\}$ **do**

$h \leftarrow$ point in s_i which maximises the area of $\triangle hlr$;

reject points bound by $\triangle hlr$;

$\{s\} \leftarrow$ all points outwards from $\overline{lh}$;

$\{s\} \leftarrow$ all points outwards from $\overline{rh}$;

endfor □

The divide-and-conquer algorithms divides a problem into sub-problems, finds the convex hull for each one of them and then merges these together by finding the convex hull of convex hulls. Algorithm 12 finds the convex hull of two convex hulls.

Algorithm 12 *Convex hull of convex hulls*

$p \leftarrow$ one point in h_1;

if p is also in h_2 **then**

$h \leftarrow$ **scan** around from p, and **merge** h_1 and h_2;

else

$(u, v) \leftarrow$ points on h_2 such that $\angle upv$ is maximised;

$c \leftarrow$ the chain from u to v which is furthest away from p;

$h \leftarrow$ **merge** c and h_1;

endif □

There are also dynamic algorithms for finding the convex hulls. In this case the input is online and one can not look ahead at the

input. This kind of algorithms may be useful for online applications, for example the traffic control in real time. Two things can happen in such dynamic algorithms; points are inserted or points are deleted.

The gift-wrapping methods, which are similar to the Jarvis's march, can be extended to the general d dimensions. Here a point is beneath a facet if it is on the same side of it as the hull, otherwise it is beyond it. Also, with respect to a point p, if p is beneath all facets that contain v then v is concave; if p is beyond the same then v is reflex; otherwise v is supporting.

Appendix 1
Bibliography

Donald E. Knuth. *The art of computer programming. V. I Fundamental algorithms.* 3rd ed. Addison-Wesley, 1997

Franco P. Preparata and Michael Ian Shamos. *Computational Geometry. An introduction.* Springer-Verlag, 1985

G. S. Rushbrooke and D. J. Morgan. On the magnetically dilute Heisenberg and Ising ferromagnetics. *Molecular Physics.* vol. 4, 1–15, 1961

Kittisak Tiyapan Computation of Fluid Flow. *M.Sc. dissertation.* September, UMIST, 1995

K N Tiyapan. Division of space by Voronoi graphs, percolation within percolation and application to the models of porous membranes. *Ph.D. Thesis.* University of Manchester, 2004

P. J. Wray, O. Richmond and H. L. Morrison. Use of the Dirichlet tessellation for characterizing and modeling nonregular dispersions of second-phase particles. *Metallography.* **16**, 39–58, 1983

Porous Media

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In one study (Grosfils *et al*, 1999) of the propagation in random Delaunay lattice particle on arriving at a site in the lattice deflects over the largest possible angle to either the right or the left, depending on the right- or the left scattering nature of the site. After the particle has passed through the site the latter goes into the reverse state. When a similar study is done on the triangular lattice, the entire trajectory quickly becomes confined to a particular strip which is bounded by two adjacent parallel lines of the lattice. They explain the propagation as being due to a blocking mechanism which prevents the particle from moving in a direction opposite to the propagation direction for more than a few steps.

The flow of fluid in porous media follows the Darcy's Law which states that the rate of flow through such a medium is proportional to the potential energy gradient within that fluid. The constant of proportionality is the hydraulic conductivity, which is a property of both the porous medium and the fluid moving through it, $v = Q/A = -kdh/dl$.

The average velocity over the entire cross section is called the superficial velocity, $v_s = Q/A = \dot{m}/\rho A$. The velocity that is based on the actual open space within the porous media is called the interstitial velocity, $v_n = Q/(\varepsilon A) = \dot{m}/\varepsilon\rho A$, where ε is the porosity, $\varepsilon = V_v/V_s = (V_t - V_s)/V_t$, V_v, V_s and V_t are respectively the void, solid and total volume.

The definition of porous media is different from that of porous materials. A porous medium is a medium through which other substance may pass, whereas a porous material merely means some certain kind of material the internal structure of which is filled with pores. Most of porous materials can be used as porous media. All of them are useful because of their internal structure, and though we do not need to know this to be able to use them, we do have to understand it if we want to use them efficiently, or if we want to improve upon some of their particular properties.

The outer bark of Quercus suber L., for example, has a property that is ideal for its commercial use as cork. Its material property is such

that it neither shrink nor expand when stretched or compressed. This makes it an excellent insulator of both heat and sound as well as good for damping vibration (*cf* Ashby, 1990). It has high frictional coefficient, and is both impervious to liquids and chemically stable. These properties make it ideal as wine corks. By understanding its internal structure and mechanism that makes it stay in the same shape when experiencing external forces, scientists have succeeded in making a synthetic material that shrink, instead of expand radially when pressed. When such material is used as cork, it can be easily pushed inside the neck of a bottle. When we release the force it will expand to its full size and fit snuggly where we placed it.

Pores introduced into a fabric can also increase its commercial value by giving it a lighter weight, higher heat retention rate, which makes it ideal for both ski and snow-board wearer and casual wear (JETRO, 2001).

Some filters have a fibrous structure, for example the dust filter studied by Bahners and Schollmeyer (1986).

The filtration coefficient λ in $-dc/dl = \lambda c$ is not a constant but change with time because of particles adsorbed by the bed. If one assumes that particles are bound to the wall by London-van der Waals force only, then $\lambda = K(\epsilon_0 - \sigma)$ where σ is the specific deposit of solids in filter bed described as volume of solid per unit filter volume, ϵ_0 the initial porosity of the bed and K a function of London-van der Waals constant, d, ϵ, η and W. The equation of continuity is $W_0(dc/dl) = d(\sigma)/d\theta$. Cake is stabilised by the flow force and consolidates when it has reached a critical thickness and the velocity dropped below a critical value. The cake pressure is highest at the interface with the filter, so it is here that it starts to consolidate. Vibration is normally used to loosen it.

Frederic Lewis (1928) in his study of cucumber cells finds that the number of sides in each of them are usually 6. These tall epidermal cells cover in a single row the rind of cucumber in a very regular manner with the average number of sides touching other cells on the same layer is 5.996. When these cells divide, the plane of division usually crosses a short axis of the polygon, which implies that the dividing partition extends from one wall of the cell to another. The number of sides and the number of vertices being equal, if we let n_0 be the number of vertices in the original cell while n_1 and n_2 that of the two cells (c_1 and c_2) thus resulted, then, because each of the new cells has two new vertices, $n_1 + n_2 = n_0 + 4$. Dividing cells have an average number

of sides 6.992. Division of a pentagonal cell invariably produces a pentagon and a quadrilateral. Sixty-eight per cent of the hexagonal cells division is into a pair of pentagons, but they could also be divided into a quadrilateral and a hexagon. A heptagon normally divides into a pentagon and a hexagon, but some five per cent of the division gives a heptagon and a quadrilateral. Octagons usually divide into a pair of hexagons, but 24 per cent of the division creates a pentagon and a heptagon, and it is theoretically possible to have a quadrilteral and an octago. All the nonagons division observed were into hepagons and hexagons. One layer of regular hexagons superimposed on top of another give a projection of tetrakaidecahedra. The underlying cells become facetted in such a manner that they may become a part of a heptakaidecahedron.

There are similar patterns of grains in a polycrystalline ceramic, magnesium oxide, cadmium, *etc*. The moments of distribution above $n = 6$ is $\mu_m = \sum_n (n-6)^N f_n$ where f_n is the fraction of cells with n sides. The second moment $\mu_2 > 0$ unless all cells have six sides, and we have a purely topological relation $\sum_n n_n^e n_e f_n = \mu_2 + 36$. If $n_n^e \propto 1/n_e$, then $n_n^e = (6 - a + b\mu^2/6) + (6a + (1-b)\mu_2)/n$. If $a = 1$ and $b = 0$, this equation is reduced to $n_n^e = 5 + (6 + \mu_2)/n$ and furthermore if $\mu_2 = 0$, $n_n^e = 5 + 8n_e$. In a polycrystal the distribution of n_e does not usually vary as the grains grow. Typically $\mu_2 = 2.4$. Soap foams resemble a polycrystal (Aboav, 1980), and $n_n^e = A + B/n_e$, $n_n^e = (6-a) + (6a + \mu_2)/n$, $a = 1.2$.

The grains of polycrystals are not arranged at random but in a characteristic way which can be expressed in simple terms and seems to be scale-free (Aboav, 1970). The average number of sides of neighbours of a grain is $n_n^e = 5 + 8/n_e$ where n_e is the number of sides of the grain. Aboav (1970) studies grains of polycrystalline magnesium oxide and finds $n_e = 5.85$. He also finds that most of the time $n_n^e > n_e$ which seems like a contradiction but is because the probability that a point lies in a grain of a particular shape depends on the size and abundance of such grains. He also finds the average number of n-sided grain $n_c^{n-e} \propto (n-2)$. Grains of a polycrystal are different from cells of a soap foam in that they possess a stable grain diameter, which only depends on the temperature.

This stable diameter of grain, d_e, is the average grain diameter at which the growth ceases, and is $d_e^{1/2} = c(T - T_0)$ (Aboav, 1971). For cadmium $T_0 = -53°\text{C}$, $0°\text{C} < T < 170°\text{C}$. The distribution of grain size is $z = z_m \exp\left\{-\alpha^2\left[(x/x_m)^{1/2} - 1\right]^2\right\}$ where z is the number of grain

sections in a plane section, α a constant, x the diameter of grain section and x_m the value of x at z_m.

The shape of individual grains in polycrystal and the inter granular surface can be found by stereoscopic microradiography, but using random plane section is more convenient in practice (Aboav and Langdon, 1969).

Gas turns into liquid by the growth of larger and larger clusters. But unlike percolation in networks, clusters in condensation process are not well defined. Crystalisation is also characterised by the growth of one phase within another. But here the orientations are an inherent part of the clusters themselves, and there is a long range coordination among clusters. One important thing in percolation is for the system size to be infinite. Random discs overlapping one another in a continuum helps correct counts of bacteria cultures (*cf* Essam, 1980). Let p_c be the critical probability and $P(p)$ the percolation probability. Then as p approaches p_c from above, $P(p) \approx (p - p_c)^\beta$ where $0.4 < \beta < 0.5$ is the critical exponent. If p approaches p_c from below, $S(p) \approx (p_c - p)^{-\gamma}$ and $\xi(p) \approx (p_c - p)^{-\nu}$ where $S(p)$ is the conditional average $\langle s|F\rangle$, s the number of particles in a cluster, and F the event that the origin O is occupied and belongs to a finite cluster. In other words, $S(p)$ is the mean size of cluster at the origin given that F occurs. This last event occurs with probability $p_F = p(1 - P(p))$ and $p_F = p$ for $p < p_c$. Furthermore, $S(p) = p_F^{-1} \sum_r C(r,p)$ and $\xi^2(p) = [p_F S(p)]^{-1} \sum_r r^2 C(r,p)$ where the pair-connectedness function is $C(r,p) = \langle \eta(r)|F\rangle p_F$, and $\eta(r)$ is the indicator defined to be one if r is connected to O and zero otherwise. When F occurs, $s = \sum_r \eta(r)$. There are two different definitions of the critical probability, $p_c = \sup\{p|P(p) = 0\}$ and $\pi_c = \sup\{p|P(p) = 0 \text{and} S(p) < \infty\}$, sometimes denoted by p_H and p_T for Hammersley and Temperley respectively. By definition, $\pi_c \leq p_c$. When p approaches p_c from above, γ' and ν' are similarly defined respectively by $S(p) \approx (p - p_c)^{\gamma'}$ and $\xi(p) \approx (p - p_c)^{\nu'}$ It is generally assumed as $\gamma' = \gamma$ and $\nu' = \nu$. No proofs for these assumptions exist, even though they are consistent with the series expansions. Estimates of γ and ν are $1.6 < \gamma < 1.7$ and $0.8 < \nu < 0.9$. In a dilute ferromagnet a cluster containing s spins each of which has a unit magnetic moment has a probability $p = \exp(\frac{1}{2}sh)/\left[\exp(-\frac{1}{2}sh) + \exp(\frac{1}{2}sh)\right]$ of being parallel to the magnetic field $H > 0$. Here $h = 2H/k_BT$ where k_BT is the thermal energy. For an infinite cluster, $p = 1$. The zero-field magnetic moment is $\mu_0(p) \propto P(p)$. The zero-field magnetic moment is $\mu_0(p) \propto P(p) \sim (p - p_c)^\beta$. The field dependent magnetic moment at p_c is $\langle (1 - \exp(-sh))/(1 + \exp(-sh))\rangle_F$ and $\frac{1}{2}(1 - G(p_c, h)) \leq \mu_c(h) \leq 1 - G(p_c, h)$ where $G(p, h) = \langle \exp(-sh)\rangle_F$. Therefore the critical probability δ is defined by $\mu_c(h) \sim 1 - G(p_c, h)h^{1/\delta}$.

The correlation function between σ on site i and j is defined by $\Gamma_{ij} = \langle\sigma_i\sigma_j\rangle_T - \langle\sigma_i\rangle_T\langle\sigma_j\rangle_T$ where $\langle\cdot\rangle_T$ is an average over spin states. Then $\Gamma_{ij} = 1$ when i and j belong to the same cluster and zero otherwise. The fluctuation formula is $k_B T\langle\chi\rangle = \sum_j\langle\Gamma_{ij}\rangle$ where $\langle\Gamma_{ij}\rangle = C(r_j - r_i, p)$, the mean susceptibility is $\langle\chi\rangle = p_F S(p)/k_B T \sim (p_c - p)^{-\gamma}$ and the mean free energy is $\langle F\rangle = (k_B T \ln 2)K(p)$ where $K(p)$ is the mean number of clusters and $K(p) \sim (p_c - p)^{2-\alpha}$, where the index assigned to the third derivative divergence is $1 + \alpha$, $\alpha \approx -0.5$.

Shape is the most fundamental geometrical property. Shape and size are all the geometrical information that remain when location and rotational effects are filtered out from an object (Dryden and Mardia, 2002), and between these two you can take the size away so that only shape remains for further analysis. When we talk about particle sizes in simulation, it is usually the case that we have already assumed some kind of particle shape. This is because the definition of size is only meaningful if you have some idea about the shape. Shape analysis works with landmarks, which are also known as anchor-, control-, design-, key-, model-, profile-, or sampling points, facets, markers, nodes, sites, fiducial markers, *etc*. Dryden and Mardia (*ibid.*) work with three types of landmarks, *viz.* anatomical-, mathematical- and pseudo landmarks. Their work could become very interesting if combined with another problem of object location (*cf* Tiyapan, 1996). A landmark can be unlabelled or labelled with a name or number. A particular member of the shape set which is used as a representative of all other members is the icon of that set. For the shape analysis in two dimensions the thin-plate spline is a convenient tool which is bijective and is analogous to the monotone cubic spline.

Bibliography

D A Aboav and T G Langdon. The shape of grains in a polycrystal. *Metallography.* **2**, 171–178. 1969.

D A Aboav. The arrangement of grains in a polycrystal. *Metallography.* **3**, 383–390. 1970.

D A Aboav The stability of grains in a polycrystal. *Metallography.* **4**, 425–441. 1971.

D A Aboav. The arrangement of cells in a net. *Metallography.* **13**, 43–58, 1980

M F Ashby. Cork. in *Encyclopedia of Material Science and Engineering.* 854–858, Pergamon, 1990

Ian L. Dryden and Kanti V. Mardia. *Statistical shape analysis.* John Wiley & Sons. 2002.

J. W. Essam. Percolation theory. *Reports on Progress in Physics.* vol. 43, 835–912, 1980.

Patrick Grosfils, Jean Pierre Boon, E. G. D. Cohen and L. A. Bunimovich. Propagation and organization in lattice random media. *Journal of Statistical Physics.* vol. 97, nos. 3/4, 575–608, 1999

JETRO. Nylon 66 Spun Fiber Fabric. *New Technology. Japan.* vol. 29, no. 6, p. 32, Japan External Trade Organisation, September, 2001

Frederic T Lewis. The effect of cell division on the shape and size of hexagonal cells. *The Anatomical Record.* **33**, 5, 331–55. September, 1926.

K Tiyapan. On an algorithm for object-location problem. (Object location using Extremum Control) *1996 Advanced Theory and Application of Control Systems.* 6^{th} –8^{th} October , Izu Recreation Centre, Atagawa Heights, Atagawa, Japan. Fu-I1–Fu-I5, 1996

Reader's Me

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In the beginning there was the Word and the Word is Me and the Word was God. But now since we can think God is the Word but the Word is not God. The Word is Me and Me is not God but God is Me. In the beginning men lived in Paradise, like all other animals which never die so long as our gene propagate through time. But then men want to know why we are here and to gain knowledge and to separate good from bad and to have science, and that is where we sin. That is where sin comes in. Also in the beginning the Me was the image of God. But since we had wanted to know we now know that God is still Me, but the Me is no longer God. That is where we also sin, when we want to know about those parts of God which are not Me. Both then and now Creation is still the evolution, but the evolution is not Creation, but now we die when our body ceases to exist since each one of us is different from the other in that we know a different amount of what parts of God are which are not Me. Before this we died only when our gene ceases to exist as a specie. Sometimes that extinction is the Creation and evolution. When one specie ceases to exist or becomes others according to Darwin's theory of the complement of Me in God.

All men are sinful, and women, because we know the difference between good and bad, and because of science. God is great but Satan is irrelevant, and because Eve made Adam eat the fruit of knowledge we have science. The mother of science Eve is ever no evil, and Faraday no hero because since we have electricity the milky way never appears to us in the city, and because of this we are miles away from God though He is here. Because Adam loves Eve we have science. Electric light comes from science, so some products of science induces sin.

Follow your heart not your head! You already have your revelation, wherefore are you looking for details? Why do you still keep searching amongst all the stories? Your revelation is yours whereas Paul's words are Paul's!

It is amazing how teachings of most of the religions and philosophies I know seem to me not merely similar but exactly the same. One may begin by looking at Christianity, Islam and Judaism, all of which has only one and the same God.

In Judaism you pray directly to God, and likewise in Islam. That is to say, you reach God by no one except yourself.

In Christianity the situation is somewhat more complicated. Here there is no single bible but two bibles, that is the Old and the New Testaments, the theme of the former being Fear whereas that of the latter Love.

In the Old Testament one may reach directly this God that one regards with fear, thus the term *God-fearing people*.

What Jesus teaches us in the New Testament amounts mainly to, 'No one comes to My Father except through Me'. If one only takes away the quotation marks, then His *Me* and *My* become conversely the reader's *my* and *me*.

Whether one is aware of it or not, this technique of a *Reader's Me* is by no means unusual in Literature. It is an enormously effective technique which may provide a good explanation to many other things, for example the saying, 'You are what you read', and why written words are usually more convincing than spoken ones.

In a way, God is our consciousness or subconsciousness.

For I have not spoken of myself; but the Father which sent me, he gave me a commandment, what I should say, and what I should speak. And I know that his commandment is life everlasting: whatsoever I speak therefore, even as the Father said unto me, so I speak. (John, 12: 49,50)

Not only does God create us, but he is here with us, everywhere and within us. One must hold true to one's faith and and believe in one's self.

And yet if I judge, my judgement is true: for I am not alone, but I and the Father that sent me. (John, 8: 16)

When I pray, I talk to God in my own mind. You do not talk to God together as a crowd but individually. In prayers lies the strength of our thought which others have no access to. They have a similar thing of their own.

But thou, when thou prayest, enter into thy closet, and when thou hast shut

thy door, pray to thy Father which is in secret; and thy Father which seeth in secret shall reward thee openly. (Matthew, 6: 6)

Jesus died, but he dies not. He lives always in us who believe in him. And since he is in his Father, similarly I am also in my Father, I who believe in Him.

Yet a little while, and the world seeth me no more; but ye see me: because I live, ye shall live also. At that day ye shall know that I am in my Father, and ye in me, and I in you. (John, 14: 19, 20)

If you believe in God, you also believe in Jesus, you also believe in yourself.

Let not your heart be troubled: ye believe in God, believe also in me. (John, 14: 1)

And as soon as we believe in Jesus's words, every *me* he uses becomes our *me*, that is it means each one of us to ourselves alone.

Jesus saith unto him, I am the way, the truth, and the life: no man cometh unto the Father, but by me. (John, 14: 6)

In Bhagavati Gītå. the king Arajuna hears voices from a god telling him first to do his duty and then to *follow no one but Me*. Following the same line of the above argument, I reason that this *Me* is not the *speaker*, that is the god, but the *listener*, that is Arajuna himself. Our hearing is in fact our repeating things we hear to ourselves. A German, following the same line of thinking but looking at things from another angle, says, 'Wie soll ich wissen, was ich sage, befor ich höre, was ich rede', that is to say, 'How could know what I say before I hear what I speak'. Buddha's teaching is exactly the same with the teaching of all the above religions, except that he puts it in a much simpler and straight-forward manner, 'No one comes to the Truth except through himself'. Amen or Sạdhu, whichever. And *quod erat demonstrandum*.

Stranded Words?

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31st July 2004

A few years ago, two words were brought up to my mind by a very respected person [2] whose profound knowledge in the English language, in my point of view, can hardly find equals. These are the words *unkempt* and *dishevelled*. Both has got a similar meaning. To be *unkempt* is to go around with your hair not combed. It means having untidy hair in milder a degree than having your hair *dishevelled*, which is extremely disorganised.

These two words are very interesting for the reason that at a first glance both appear to be prefixed words, that is *un* + *kempt* and *dis* + *hevelled*. This implies that it should be fine to make the sentences "the groom is both well dressed and kempt today", and "before going to school, hevel your hair first!" (for example, from a mother to her son). But unfortunately there is no such verb "to hevel" in English, neither is there an adjective "kempt". In fact, after a little effort of searching we can find that neither of them is associated to any other English word. Let us take a look into each of these, one by one.

Let us look at the word *unkempt* first. This word can be traced back to a Latin word *incomptus* meaning "unadorned" or "untrimmed" [1, 3]. Similar words include, *úkembdr* in Old Norse, *cemban* in Old English which means "to comb", *kembian* in Old Saxon, *kemben*, *chempen* in Old High German, *kemba* in Old Norse, *ungemembet* in Middle High German, *kämmen* in German, *kambjan*, *kambaz* in Germanic, and *incontos* in Italian which means "unadorned".

After that let us look at the word *dishevelled*. The Latin word *capillus* (from which came *capillary*) means "the hair of the head" gave rise to an Old French word *chevel*. *Chevel* (Old French) can be prefixed to give *descheveler* with its Past Participle *deschevelé*, which in turn produced *dischevel*, *dischevelee* of late Middle English (Chaucer uses *dischevely*). The Past Participle form *dischevele* in Middle English means "with hair in disorder" [4].

§ Reference

[1] *Oxford dictionary of English Etymology.* Oxford. 1966.

[2] Amnad Khitapanna. private communication.

[3] C. T. Lewis and C. Short. *A Latin dictionary.* Oxford University Press. 1951.

[4] A. L. Mayhew and W. W. Skeat. *Concise dictionary of Middle English.* Oxford University Press Warehouse.

Electric Car, if not this, then what else?

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Abstract

Over one century has passed since we first began to use an electric car. Have we become mature enough to cope with it now, or will it again prove a dead technology? This article gives a brief description of the state we are in now regarding electric vehicles. It also discusses fundamental issues and lays down directions necessary for this multi-disciplined project.

Introduction

Battery-powered motor vehicle was invented first in the late 1880s in the form of car, truck and bus. They had until 1920 been competitive with petroleum-fuelled car. Except perhaps for their brief return in the US in 1960s due to an oil crisis, they have largely disappeared from the automobile production lines.

We may prolong to a certain degree our exploitation of the fossil fuel by getting the most of it possible from underground and by using it more efficiently. A large part of oil in the reservoirs permeates and is retained inside porous rocks. There have been studies to understand better the nature of this retention, which could help us delay the depletion of oil-fields. This is one application of the bond problem (*cf* Dullien, 1979; Scheidegger, 1960) of percolation theory (*cf* Stauffer, 1985; Tiyapan, 2002 and 2004.) To use oil wisely we may on one hand produce cars which have better power-consumption efficiency. On the other hand we may improve our road networks and traffic controls such that to minimise the overall congestion. One example of this latter is the idea based on the symmetry of percolation thresholds suggested by Tiyapan (2002, appendix.)

All hydrocarbon or fossil fuels when burnt produce carbon dioxide that acts as a blanket which lets the heat from the sun to enter the Earth's atmosphere but prevents it from radiating away at night. This is known as the *greenhouse effect* by the result of which the earth is warming up, disturbing the global balance of the weather. This is one reason why the research for an efficient electric car is important.

History

Alessandro Volta invented the primary battery in 1780, Plante invented the secondary battery in 1860. The former is known as a one-way battery whereas the latter may be recharged and reused. Volta's battery used stacks of different metals separated by cloths which are moistened with an ionic solution. Plante's electric cells were essentially a lead-acid system.

In 1881 M. Gustave Trouvé demonstrated for the first time his motorised tricycle powered by lead-acid batteries. By late 1880s battery-powered motor vehicle was used for private passengers as well as for truck and bus. The first electric four-wheeler was made by William Morrison in 1891. In 1899 a four-wheeled electric car made by Camille Jenatzy became the first ever to break the 60 mi/hr record. Most taxis in Berlin, Boston and New York were electric. London in 1910 had registered some 6,000 electric cars and 4,000 commercial vehicles.

With the help of people like the inventor and industrialist Elmer Ambrose Sperry (1860–1930), in its early stage it was competitive with petroleum-fuelled car. Then the progress in the development of the latter became faster, for example the Kettering electric self-starter which was used in 1912 Cardillacs. By the end of the First World War the use of electric propulsion had become limited to wayside power-trains, trams and trolley bus. These were gradually replaced by the diesel-engined bus. In London the tram lasted until 1952, and the trolley bus until 1962. After 1920 electric car was largely forgotten, to reappear briefly in the US during 1960s due to an oil-crisis, but disappeared again by mid-1980s.

In 1990 the California Air Resources Board talked about zero-emission vehicles (ZEV, by legal definition in the US the electric vehicle is a ZEV), and about imposing the requirement that all car-manufacturers produce a certain amount of these. Unfortunately the idea was later dropped.

In Thailand hybrid buses have been tested and successfully used. One example is the route-36 EURO-II diesel bus which derives its energy from two sources, namely a battery system and a diesel-powered generator set.

Possible implementation

Count Felix Carli in 1894 devised a system of rubber springs that give additional power to his electric tricycle. Although such hybrid vehicles are also a possibility, even a somewhat more probable one, we do not think it is the right approach to the problem and therefore shall not consider it here. For one thing such hybrid systems are usually more complex and costly. We believe a simplest system is the best. Regarding hybrid possibilities Müller (1977) gave a good overview of combined energy sources and storages. The storage for mechanical energy, for instance, could be done using the fly wheel.

Using palm-oil to power a diesel engine has been studied and satisfactorily tested by other researchers, for example the team led by his majesty the king of Thailand. The development and success of these researches are both interesting and exciting. But our scope is only limited to electricity and battery. The latter stores the former, the former runs the vehicle.

Solar energy can be used to supply the energy for recharging the batteries. This can be in the form of either a charging dock at home, or a recharging facility at the battery-exchange station. Yamamoto (1994) called the second half of the twentieth century the *era of semiconductor*. Developments in the field of semiconductor and solar cell result in the utilisation of solar energy becoming more and more feasible. Carbon in the form of artificial diamond is used as heat-absorbing material to absorb the heat generated by the current flow in semiconductor. There are also other carbon structures, namely the Buckminster Fullerine C60 and C80 the application in this field of which needs to be investigated. The Earth receives 10^{18} kWh of solar energy each year, while our overall energy consumption is annually less than 10^{14} kWh. So theoretically at least the solar energy still looks well as a renewable source of energy for the electric car of the future.

Problem and solution

The lead-acid batteries used in most electric cars are heavy, and their capacity are reduced in cold weather. Charlesworth (1977) argues that one tonne of lead acid traction batteries contains the same amount of energy as one gallon of petrol, thus most of the energy expended in just moving the fuel about. According to Plust (1977) the energy density of batteries is 60–200 Wh/kg. Examples of other types of battery apart from the lead-acid batteries are nickel-zinc-, nickel-iron-, metal-air- and zinc-air batteries.

With regard to this problem, electric cars are still waiting for some

breakthrough similar to those which happened for the internal combustion engine. There have been efforts to develop a better electric cell, an example of which is the US Advanced Battery Consortium that was formed in 1991. Since in electric car the battery must supply all the energy needs, that is to say, the propulsion power as well as lights and other accessories, for example air-conditioning and radio, it is arguably the most important criterion for the success of the project.

There are presently no battery charging and exchange stations. We not only must have these but also need to have them everywhere, considering the limited ranges of electric car at present. While it is possible to add these facilities into existing petrol stations, there are still some safety issues to consider.

If we use fuel cells instead of a battery it may be easier to make changes to to petrol stations to accommodate servicing facilities for these. Liquid methanol and gasoline can be pumped into a car which are later converted by a reformer on board into hydrogen, which then become the fuel.

Steps to take

Dry cells are much lighter than rechargeable batteries. The possibility, feasibility and environmental consideration of using dry cells instead of rechargeable cells should be investigated.

As for secondary batteries efforts have been made to develop better configurations from iron-air-, lithium-chlorine-, lithium-sulphur-, nickel-cadmium-, silver-cadmium-, silver-zinc-, sodium-sulphur- and zinc-air systems (*cf* Bayliss, 1977.) Study for better batteries needs to be carried out further. Also important are researches on superconductor.

The use and design of solar cell to harvest energy from the Sun need to be considered. This includes among other things the on-board charging facility. The development of electric car should go hand in hand with the research for a better solar-energy technology.

One important step to take is to develop a control and information system for the traffic of electric vehicles. An example of this is the Intelligent Vehicle/Highway Systems that has been proposed, which sits on the infrastructure of telecommunication networks.

We plan to investigate also the possibility of powering the electric car externally, in a way similar to the overhead power-line and pantograph which supply railcars. For this the power supply must be integrated into all roads. Instead of having all the roads powered we could have recharging nodes at intervals, possibly at intersections in the city and service stops along interurban roads.

Instead of container tanks, such system would rely on transmission and distribution lines. Ultimately this leads to the question of where to find the energy to supply the extensive networks of power-supply which is needed. The answer to this question I think lies in the future of wind energy, solar arrays and nuclear fusion generation. These are also under study.

Conclusion

Renewable sources of energy are much needed in the foreseeable future. Extinction of species and termination of past civilisations were equally likely to be caused by a disappearance of energy sources as by other catastrophic means. A major use of the fossil fuel is for transportation, therefore the development of electric car is very important in order to address this coming energy crisis. However, the research toward this end is multi-discipline. It involves among other things the study of batteries and the use of nuclear-, solar- and wind energy to supply the energy for the batteries.

Bibliography

Bangkok metropolitan electric bus testing project. Final report. SKI Electric. The Pollution Control Department, Minister of Science, Technology and Environment. Sept, 2000.

D Bayliss. Electric vehicles – Can they be fitted into urban Britain? *International Conference on Electric Vehicle Development.* 31st May–1st June. Electric Vehicle Development Group.37–45. 1977(1978).

G Charlesworth. The energy and resource implications associated with the widespread use of electric vehicles. *International Conference on Electric Vehicle Development.* 31st May–1st June. Electric Vehicle Development Group.6–13. 1977(1978).

W H Crouse and D L Anglin. *Automative mechanics* 10th ed.Glencoe, Macmillan/McGraw-Hill. 1993(1946).

F A L Dullien. *Porous media fluid transport and pore structure.* Academic Press, London. 1979.

Encyclopaedia Britanica. 15th ed. 1988.

McGraw-Hill Encyclopedia of Science and Technology. 9th ed. 2002.

H –G Müller. Road vehicles with combined, at least partly electrical driving systems and energy supplies. *International Conference on Electric Vehicle Development.* 31st May–1st June. Electric Vehicle Development Group.48–52. 1977(1978).

H G Plust. Recent developments in power sources with special emphasis on alkaline batteries. *International Conference on Electric Vehicle Development.* 31st May–1st June. Electric Vehicle Development Group.14–24. 1977(1978).

A E Scheidegger. *The physics of flow through porous media.* University of Toronto. 1960.

D Stauffer. *Introduction to percolation theory.* Taylor & Francis, London. 1985. (2nd ed. 1992, 2nd revised ed.1994, 1998).

K Tiyapan. *Percolation within percolation and Voronoi Tessellation.* Kittix, Bangkok. 2002.

K N Tiyapan. *PhD Thesis.* UMIST, UK. 2004.

Shigeo Yamamoto. *Sō̤rå̇. denchi no hanashi.* Nikkan Kō̤gyō̤ Shimbun. 1994. (in Japanese) And the English translation by the author, *Solar source system.* June, 1999.

Percolation within Percolation Theory

Kit Tyabandha, Ph.D.

Forces between particles

Essential to the simulation of stochastic models of particles is the consideration of forces acting on each particle (*cf* Schumacher, 1996). Since in theory the forces acting between two particles extend their effects to infinity, one has to make approximations. The degree of justification to these approximations depends on how the force in question vary with distance.

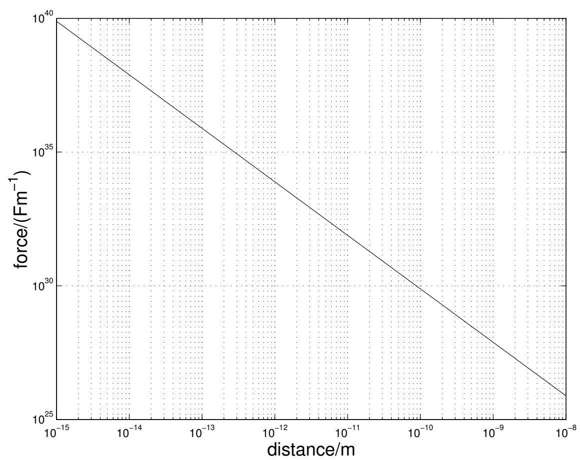


Figure 1 *Coulomb force vs distance in log-log scale*

In the case of the Coulomb force where the potential energy is $u(r) = q_1 q_2/(4\pi\epsilon_0\epsilon_r r)$ and the force $f(r) = -du(r)/dr$ is

$$f(r) = q_1 q_2/(4\pi\epsilon_0\epsilon_r r^2),$$

the justification is high as is seen in Figure 1 where the force-distance is a decreasing straight line on the log-log graph, q_1 and q_2 are protons, and r the distance in metre.

The effects of the Coulomb force are prominent when sizes of the particles are small. In quantum mechanics where the scale is atomic, for instance, the problem becomes one in which there are a large number of particles interacting with one another, *i.e.* an n-particle problem. In theory the Hamiltonian operator can be applied and then the problem solved numerically. But in general this is not possible due to the too many particles involved in the calculation. Theoretical solution is possible by the various methods of approximation, for example the Hartree method (*cf* Brown, 1972) where the best wave function is found in terms of the one electron function, *i.e.* orbitals, Φ, or a Hatree-Fock approximation which reduces the number of equations to $n/2$.

Problem definition

Walls in a 3-d Voronoi tessellation are isomorphic to bonds that link between its cells. Therefore we can redefine the problem of a particle passing through the hole in a wall, into the cell chamber and out through a hole in another wall, and so on, as that of the passage of a mathematical particle through a tube that links between two cells, into a cell and out through one of the other tubes. In reality the solid parts which make up the partitions have thickness, therefore the holes through the walls will have a nonzero thickness and the volume of the cell chamber is smaller than that of the otherwise mathematical Voronoi cell.

Each vertex of a Voronoi polygon has three walls of the latter attached to it. The centre of gravity † of these three faces is calculated and linked together. This truncates the coigns and produces for each polygon its dual self. The next step is to link together the midpoints of all those edges of the polygon that have a vertex in common. The edges of this last polygon bound and define the void volume of the cell chamber.

Next, the cross section of the hole within each wall is taken to be the area on the original wall which is bound on all sides by the second order covering lattice of that wall.

The filtering membrane in this case is assumed to be isomorphic and homogeneous. The size of the particles is taken to be reasonably smaller than the void in general, so that the attraction between particles and the adsorption to the wall play a more prominent role than

† aka centre of mass, centroid.

the physical blockages by individual particles. The particles are all assumed to be of the same size.

Algorithm 1 is an algorithm to do filtration that is being developed. Here N contains the neighbourhood information

Algorithm 1 *Filtration in Voronoi tessellation*

find $(v_a, c_a) \leftarrow$ Voronoi tessellation;

find $d_l \leftarrow$ Delaunay triangulation;

find $N_c \leftarrow$ cell neighbours;

$c \leftarrow c_a$ which lie completely within $[0, 1]^3$;

$v \leftarrow v_a$ which belong to some c;

□

My work on filtering membranes results in another Voronoi algorithm and programme which is different from the programmes for the same purpose used elsewhere. Because it is also written anew from scratch, this new algorithm has nothing to do with the previous two as regarding the data structure and the logic of its method.

The current convention for variables is this. A single alphabet or entity means a list, for example `c` is the cell list and `va` is the list of all vertices original created. Another example is b, a structure for bonds which contains the list of the cells of bonds, the number of vertices of the face represented by each bond, the list and the map of these vertices, the ordered list of the vertices of each face and another list similar to this but cyclic, and the number of cells connected to each bond. Similar to the structure of `b` is that of `bdr`, or b_d in Algorithm 2, which is a list of the bonds along the border, each of which is connected to only one cell.

Two entities xy makes x of y, for instance `vc` is vertices of cells, which is a structure that contains the number of vertices of each cell and the list and the map of all these vertices. I try to preserve the space

by creating a short but easy to understand naming convention. Also, the lines are put together, which means that the number of lines of the codes is more than what appears here, some of the lines having six logical lines or more to them. But the structure of the programme still remains intact, therefore it should be possible without much difficulty to compare the programme, which shall be published elsewhere, with Algorithm 2 which explains it.

Three entities xy^2 or y_x^2 are $y \times y$ matrix of x, where the latter relates two entities of the former, for example `bcc` is the bonds mapped on to a cell matrix. In the algorithm, v_n and c_n are respectively the `vin` and `cin` in the programme, the `vn` and `cn` there being verbosely described in the algorithm as the *number of* v and c.

Algorithm 2 *Voronoi data structure for the study of membrane filters.*

$(v_a, v_c^a) \leftarrow$ **find** a 3-d Voronoi tessellation;

$v_n \leftarrow$ index of $0 < v_a < 1$;

$c_n \leftarrow$ index of c all the v's of which are in v_n;

$v \leftarrow v_a \in v_n$;

$v_c \leftarrow v_c^a \in c_n$;

$t_a \leftarrow$ **find** a 3-d Delaunay tessellation;

for all $t_i \in t_a$ which are connected to two cells **do**

  $t \leftarrow t_i$;

else

  $b_d \leftarrow t_i$;

end

$b \leftarrow c_b^2 \leftarrow t$;

$b \leftarrow b_d$;

find Delaunay triangulation in $(3-1)$ dimensions for all faces;

order the vertices of each face; ▫

Molecular dynamics

The Lennard-Jones function can be written $u(r) = -A/r^6 + B/r^{12}$, where A and B determine respectively the attractive and repulsive parts. Let the range parameter $\sigma = (B/A)^{1/6}$ and the energy parameter $\varepsilon = A^2/4B$, and the equation becomes $u(r) = 4\varepsilon\left[(\sigma/r)^{12} - (\sigma/r)^6\right]$. In other words, σ is the diameter of one of the atoms and ε is the well depth, *i.e.* energy constant. The force here is not $F = -du/dt = (24\varepsilon/r^2)\left[2(\sigma/r)^{12} - (\sigma/r)^6\right]$, but $F = -du/dr$.

Another approach in molecular dynamics simulation is to use numerical methods on the Newton's equation of motion. The most widely used is perhaps the Verlet algorithm, shown here as Algorithm 3. The initial values are r_0 and r_1.

Algorithm 3 *Verlet algorithm, cf L. Verlet (1967)*

for $i = 1$ to n **do**

  find f_i;

  $r_{i+1} \leftarrow 2r_i - r_{i-1} + f_i \Delta t^2/m + \mathrm{O}(\Delta t^4)$ **and**

  $v_i \leftarrow (r_{i+1} - r_{i-1})/(2\Delta t) + \mathrm{O}(\Delta t^2)$.

endfor ▫

There are variants and modifications of Algorithm 3, for example the Leapfrog Verlet algorithm which has three steps instead of two, that is $v_{n+1/2} = v_{n-1/2} + f_n/m\Delta t + O(\Delta t^3)$, $r_{n+1} = r_n + v_{n+1/2}\Delta t + O(\Delta t^4)$ and $v_n = (v_{n+1/2} + v_{n-1/2})/2 + O(\Delta t^2)$; or the velocity Verlet algorithm where $r_{n+1} = r_n + v_n\Delta t + f_n\Delta t^2/(2m) + O(\Delta t^3)$ and $v_{n+1} = v_n + \Delta t(f_{n+1} + f_n)/(2m) + O(\Delta t^3)$, or in the form normally used in practice in which $v_{n+1/2} = v_n + f_n\Delta t/(2m)$, $r_{n+1} = r_n + v_{n+1/2}\Delta t$ and $v_{n+1} = v_n + f_{n+1}\Delta t/(2m)$.

The van der Waals force is an attractive force acting between molecules. In the case of gases, each molecule consumes some space, so the dynamic volume is less than overall volume by an amount bn when b is a constant and n the number of molecules. Here b is $N_A v$, where v is the volume slightly larger than the volume of each molecule of gas and N_A the Avogadro number, $N_A = 6.022 \times 10^{-23}$ mol^{-1}. The pressure of gas we see is the pressure of gas detected, which is less than the real pressure by F/A, where $F = \sum F_i$ and $F_i = \sum f_j$. Here i runs from 1 to n and j from 1 to $(n-1)$ in the same set of molecules. The reduction of the total force comes from all molecules, but this reduction from each molecule is in turn affected by those molecules around it. Therefore $i \neq j$ and j runs from 1 to $(n-1)$ as mentioned above. Suppose that each molecule attracts another molecule by a force a. Then, in a given volume V, $F = \sum F_i = \sum_i \sum_j f_{ij} = \sum_i (n-1)a/V = n(n-1)a^2/V^2$. For very large n we can say that $(n-1) \approx n$, and therefore $(P + an^2/V^2)(V - bn) = nRT$.

The nature of this mutual attraction between molecules, which reduces the pressure in gas, comes to light in the case of solids. The distribution of charges around a neutral atom in solid fluctuates in the time scale $\tau < 10^{-16}$ s. This charge imbalance makes each atom behave as an electric dipole. This electric dipole has an electric field around it, which affects other atoms nearby and binds the two together.

The electric dipole moment is $p = qa$, where a is a vector from the negative to the positive charge. The electric field around a charge q_1 has a magnitude $E = q/(4\pi\varepsilon_0 r^2)$. It acts on a nearby charge q_2 with a force of magnitude $F = u = q_2 E = q_1 q_2/(4\pi\varepsilon_0 r^2)$, where u is the potential energy of the two charges. This force itself is the Coulomb force, and the electric potential around q_1 is u/q_2, that is $V = q_1/(4\pi\varepsilon_0 r)$.

For an electric dipole,

$$V = \left(\frac{1}{(4\pi\varepsilon_0)}\right)\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q(r_2 - r_1)}{(4\pi\varepsilon_0 r_1 r_2)}$$

Since $a \ll r$, a being of the order of the atomic size, $r_1 r_2 \approx r^2$. Let θ be the angle between a and r. Then, also since $a \ll r$, we have $r_2 - r_1 \approx a\cos\theta$. Therefore we now have $V \approx qa\cos\theta/(4\pi\varepsilon_0 r^2) = p\cos\theta/4\pi\varepsilon_0 r^2$ and consequently $E_r = -\partial V/\partial r = 2p\cos\theta/(4\pi\varepsilon_0 r^3)$ and $E_\theta = -\partial V/(r\partial\theta) = p\sin\theta/(4\pi\varepsilon_0 r^3)$.

We neglect E_θ for long-range interaction, and simplify E_r to $E_r = a/r^3$. We shall hereafter use the terms *dipole* and *atom* interchangeably.

If this electric field is produced by atom p_1, it could induce in another atom $p_2 = \alpha E_r$, where α is the molecular polarisability of the second atom. This second atom will have an energy of interaction with the first one, $u = -p_2 E_r = -\alpha E_r^2 = -\alpha a^2/r^6$. This force is attractive.

The repulsive force is more complicated, and is generally thought to arise from the Pauli exclusion principle and the Coulombic repulsion of the electrons in the outer orbit. It is generally assumed to be either $u \approx A/r^{12}$ or $u \approx a\exp(-r/\rho)$, where ρ is a range parameter (*cf* de Podesta, 2002).

The Lennard-Jones potential is often written as

$$u = -4\varepsilon\left[(\sigma/r)^6 - (\sigma/r)^{12}\right]$$

where σ is a range parameter that indicates the approximate size of an atom and ε an energy parameter that indicates the strength of the interaction between atoms. In other words, it is the minimum value of u.

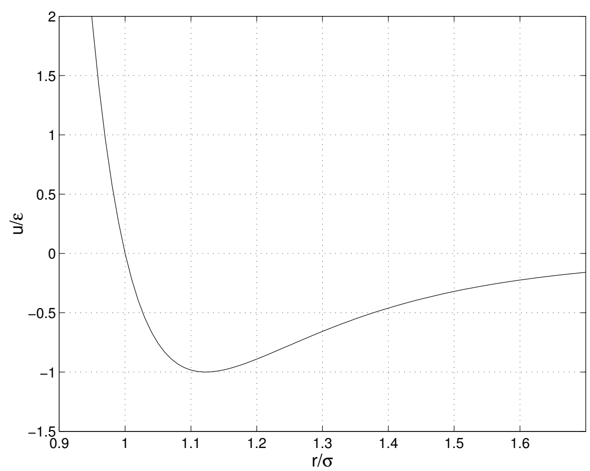


Figure 2 *Lennard-Jones potential*

The Lennard-Jones potential between two atoms has the minimum value $u = -\varepsilon$ at $r = 1.1225\sigma$. Figure 2 is a plot $y = -4(1/x^6 - 1/x^{12})$, where $y = u/\varepsilon$ and $x = r/\sigma$. The force which the potential curve of Figure 2 acts on another particle is shown in Figure 3. The force produced by a dipole is calculated from $F = -du/dr = 24(\varepsilon/\sigma)[(\sigma/r)^7 - 2(\sigma/r)^{13}]$. But in our units $y = u/\varepsilon$ and $x = r/\sigma$ above the equation becomes $y = 24(1/x^7 - 2/x^{13})$, which is plotted in Figure 3.

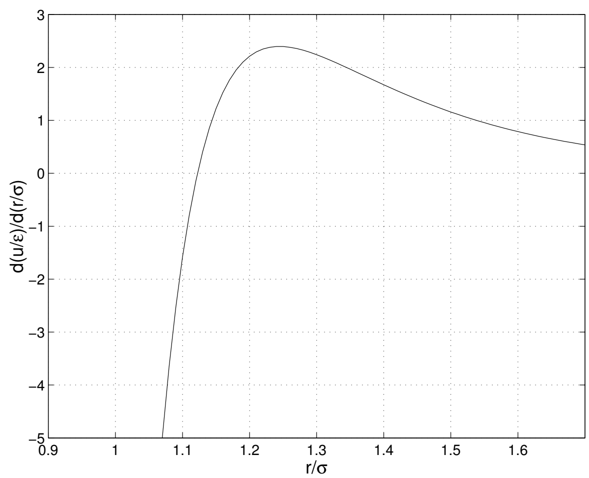


Figure 3 *Force corresponding to the Lennard-Jones potential*

Figure 3 shows that the force is zero at the distance $r = 1.1225\sigma$ away from the particle. Closer than this point the repulsive force increases rapidly towards infinity. Further away from this point, however, the attractive force increases to a maximum and then gradually dies down. This maximum force occurs when $d^2y/dx^2 = 0$, where $y = u/\varepsilon$ and $x = r/\sigma$, that is at $r = 1.2445\sigma$.:

In gases, the molecules travel past one another so fast that they hardly notice the wells of negative energy surrounding other molecules, let alone stop and rest there. But even here the attractive force produced by these wells is probably what give rise to the term an^2/V^2 in the van der Waals equation. Solid molecules, in contrast, hardly have kinetic energy and therefore prefer to sit in such wells of their neighbours, which is the reason why solids hold and neither flow as liquid does nor disperse like gas.

The most cohesive structures of solids are crystals, where the Lennard-Jones potential culminates in a minimum cohesive energy of the lattice. This cohesive energy is $U = N_A u_i/2$. Here u_i is the summation over all pair potential energies of Lennard-Jones type, $u_i = \sum_{i \neq j}^{N_A} u(r_{ij})$. Let $r_{ij} = \alpha_{ij} r_0$, where r_0 is the nearest neighbour distance. Then $U = -2\varepsilon N_A \left[(\sigma/r_0)A_6 - (\sigma/r_0)A_{12}\right]$, with the lattice sums being $A_6 = \sum(1/\alpha_{ij}^6)$ and $A_{12} = \sum(1/\alpha_{ij}^{12})$. These lattice sums are calculated from the network structure in terms of infinite series that converge very quickly.

To find the optimum value of σ/r_0 we shall let $b = \sigma/r_0$ and write differentiate $dU/db = -2\varepsilon N_A(6b^5 A_6 - 12b^{11}A_{12}) = 0$. As a result, $r_0 = \sigma(2A_{12}/A_6)^{1/6}$, the cohesive energy per mole $U = -(A_6^2/2A_{12})N_A\varepsilon$ and the cohesive energy per molecule $u = U/N_A = -(A_6^2/2A_{12})\varepsilon$.

The first algorithm we shall now develop is a reasonably simplified one. But it turns out to my surprise that this simple algorithm links us to the continuum percolation of hard spheres or even of particles with irregular shapes. This is because of the assumptions that we shall make.

One such assumption is that the variance of the filter cell size distribution is small. For this, Jafferali (1995) would probably have applied his favourite constraint code on a Voronoi tessellation. The approach which I developed uses the operator $\mathcal{G}(\cdot)$ on a $\mathcal{V}$. It is more logical and therefore is what we shall use for now.

This centre of gravity operator $\mathcal{G}(\cdot)$ has been described (Tiyapan, 2004) together with the algorithm for the case of two dimensions. In the present three-dimensional case for each polyhedron we find c.g.'s of all its tetrahedra instead of polygon those of all its triangles. Once all these centres of gravity have been found we can then add them all up in a similar manner done before. Algorithm 4 tells us how this is done.

Algorithm 4 *Centre of gravity of three-dimensional Voronoi Tessellation*

for each polyhedron **do**

  find *the centre,* which is the average coordinates of all the vertices**;**

  for all faces **do**

    triangulate the face**;**

    for each triangle **do**

      find centre of gravity of the tetrahedron made by

        this triangle and *the centre***;**

    endfor

endfor

find the centre of gravity of the polyhedron from the c.g.'s of all its tetrahedra**;**

endfor □

One favourite constraint that has been related to a Voronoi Tessellation in application is that which limits the minimum distance between any pair of the nuclei. The reason given for this is usually that in nature cells are of similar size and shape. This implies that the nuclei never get together closer than a certain distance, which can be found empirically by physically making some observations and measurements. The cause of this is often put to the hydrostatic pressure inside the cells, which pushing at the cell wall against the pressure outside at the same time pushes the nucleus back, away from them. For this reason a rule of thumb is created as a criterion used in the generation of Voronoi tessellations for the study of various kinds of structure.

Centre of gravity is a very important point for each object. The stability of a double-decker is determined not by its height but by the location of the c.g. All forces acting on an object with no angular velocity can be represented by an imaginary force passing through the c.g. In martial arts when it comes to a face-to-face combat you want to keep your eyes on the c.g. of your opponent. The limbs and anything may fly all over the place but wherever his centre of gravity be ultimately that is his move. Though in this last case one has to keep in mind the body is nonrigid, and each of its components has a different momentum.

In doing the simulation, a Voronoi tessellation is first created using Poisson points as nuclei. Then for each cell the centre of gravity is found. And then another Voronoi tessellation is created using all these centres of gravity calculated as nuclei. Repeating this procedure, the centre of gravity function $\mathcal{G}(\cdot)$, n times eventually gives us the Voronoi tessellation $\mathcal{V}(\mathcal{G}^n(\mathbf{x}))$ we want. Here $\mathbf{x}$ is the set of all the nuclei.

The use of centre of gravity this way is new and has not been found in literature. However it is simple while at the same time provides a logical starting point, a reasonable basis to work upon which can satisfactorily replace the arbitrary limiting distance criteria in use.

The second assumption is that the particle size is reasonably smaller than the size of the smallest void of the original system. This is the assumption which seems to link us to the percolation of spheres in continuum, the space considered being that of each void.

Also, we assume that the particles are attracted towards one another and towards the walls by various interactions which may include the van der Waals force and the force from electrostatic interactions.

Let θ be the angle that the line from the c.g. of a each cell to the mid point of a face makes with the horizontal plane. Then another possible assumption is that all particles prefer a trajectory which goes through a face which has the maximum θ.

The third assumption is that the volume of each membrane pore is 80 per cent the volume of the corresponding Voronoi pore. The fourth assumption, blockages due to the clustering caused by the attrition forces between particles can occur within pores. This excludes cake formation and blockage at entrances to, or exits from the pore.

The last assumption above is similar to assuming that clustering due to attrition can occur in stagnation regions. Since the flow of the suspension reaching the membrane is strong, *i.e.* the pressure high, there can be no clustering there. The flow from one pore into another is also faster than the flow within one. This is amount to assuming that the effective diameter of the hole within each face of a pore is considerably smaller than its diameter.

Upon reaching the membrane, each particle in the troop seeks out the bond closest to it, and begins its journey through the membrane. Within the membrane, it always follow the bond which has the greatest gradient available. Therefore the top bonds should map to the bottom ones one to one. But because the possible blockages during the course of operation, this may not be so and the mapping is instead one to many.

First we consider the case in two dimension, so volumes becomes areas and the volumetric flow rate the distance travelled. In our discrete time, if v is the flow velocity, n the number of particles, ρ_v the total volume ratio of the particles and r their radius, then we have $n\pi r^2 = v\Delta t$. Assuming that $v = 1$ ms^{-1} and $r = 100$ μm, then $n = 10^3\rho_v/\pi$ or approximately $318\rho_v$ particles for each time step.

Next we will study the suspension in a square box in order to find out ρ_v. Because we shall assume that the particles flowing through the pores percolate as though there is no flow, we will consider here the suspension that is simply contained within our box without moving about. I feel that the assumption that particles could percolate that way in pores is justified since the flow is in steady state, protected from the turbulence outside by all the solid structures making up the walls of the membrane.

The percolation programme first generates random positions of the particles, then moves apart those which are too close together so that in the end they only touch each other. Particles which are separated by a distance less than $1.4r$ move towards each other until they touch. Lastly, touching particles never separate.

We know that the total volume ratio of the particles has the upper limit of 0.9069, because that is the density of the closest packing of circles on a plane.

Packing circles on the plane becomes densest when the circles are arranged as a hexagonal lattice with the packing density of $\pi/2\sqrt{3}$. Packing of spheres is similarly at its highest density if the arrangement is that of the face-centred cubic lattice with the packing density $\pi/3\sqrt{2}$.

Filtering problem when physical blockage is prominent

Assuming particles to be spherical, the size of particles to be constant, and that this size is compatible with the size of the holes in the walls and the voids so that physical blocking is responsible for most of the blockages in the membrane. The hole in each wall is taken to be the polygon which results from recursively finding a covering polygon for the face twice.

Redefine the problem of particles' passage through walls and voids as that of particles travelling along edges of the dual lattice of the Voronoi structure, *i.e.* the Delaunay triangulation. Each of these edges corresponds to a face in the original physical lattice. To each edge is thus ascribed the details of the cross section of the hole through that face, namely the shape and the dimension, as well as the gradient it makes with the horizontal plane. Upon reaching a vertex, the particle ball will choose the next path, *i.e.* bond in the dual lattice, which has the maximum gradient to pass through. However, if this bond is too small or if it is blocked, the particle with choose from among the remaining

paths the one which has the maximum gradient, and so forth. If the size of the particle is approximately that of the hole it tries to pass through, within two per cent of the latter, say, then the particle will blind the passage at that point. The difference between blinding and blocking is in the degree of tightness that the particle sits in the hole, which reflects in the degree of difficulty to remove it by backflushing. This degree is not constant but a function of the relative size between the two parties involved, *i.e.* the particle and the hole.

Filtering with very small particles

At this point a new filtering algorithm is considered and investigated. We introduce an assumption that the particles are all of the same size which is very small compared with the size of the pores. So there is neither cake formation nor blinding by a single particle. Assume that the solid particles suspended in a fluid medium, being dragged downwards under their own weight.

Assume that the van der Waals force plays a significant part and that these interactions between particles are governed by the Lennard-Jones equation. Assuming that each particle is spherical and acts as a dipole with the electric dipole $p = 10e$, where e is the electric charge on a proton, $e = 1.602 \times 10^{-19}$ C. Then (*cf* de Podesta, 1996)

$$E_r = \frac{p}{4\pi\varepsilon_0 r^3}, \tag{1}$$

and the energy of interaction between two particles becomes

$$u_r = -\alpha E_r^2 = \frac{-\alpha p^2}{(4\pi)^2\varepsilon_0^2 r^6}, \tag{2}$$

where α is the molecular polarisability of one particle under the influence of the other. If we assume that the repulsive force acts in such a way that the repulsive energy is $u = c/r^{12}$, where c is a constant, the Lennard-Jones potential becomes Equation 3.

$$u_r = -\frac{\alpha p^2}{(4\pi)^2\varepsilon_0^2 r^6} + \frac{c}{r^{12}}. \tag{3}$$

Comparing Equation 3 to the form

$$u_r = -\frac{A}{r^6} + \frac{B}{r^{12}}, \tag{4}$$

we have $A = \alpha p^2/((4\pi)^2\varepsilon_0^2)$ and $B = c$. Or if we compare it to the form

$$u_r = -4\varepsilon\left[(\frac{\sigma}{r})^6 - (\frac{\sigma}{r})^{12}\right], \tag{5}$$

then we have $\sigma = (B/A)^{1/6} = (16c\pi^2\varepsilon_0^2/(\alpha p^2))^{1/6}$ and $\varepsilon = (A^2/4B) = \alpha^2 p^4/(4c(4\pi)^4\varepsilon_0^4)$. Notice that here σ and ε are simply parameters, not the Stephan-Boltmann constant and the dielectric constant, σ is a range parameter and approximates the size of an atom while ε is an energy parameter which shows the strength of the interaction between particles. At $r = \sigma$ the value of u_r is zero, whereas u_r has the minimum value of $-\varepsilon$.

The force between two particles is Equation 6.

$$F = -\frac{du_r}{dr} = -\frac{6\alpha p^2}{(4\pi)^2\varepsilon_0^2 r^7} + \frac{12c}{r^{13}}. \tag{6}$$

At the equilibrium separation, r_0, $F = 0$ and therefore Equation 6 yields the minimum distance in Equation 7,

$$r_0 = \left(\frac{32c\pi^2\varepsilon_0^2}{\alpha p^2}\right)^{\frac{1}{6}}. \tag{7}$$

The equation for the minimum energy is obtained by substituting r_0 from Equation 7 into Equation 3, which gives

$$\begin{aligned} u_r &= -\frac{\alpha p^2}{(4\pi)^2\varepsilon_0^2 r^6} + \frac{c}{r^{12}} && (8)\\ &= \frac{1}{r_0^6}\left(-\frac{\alpha p^2}{(4\pi)^2\varepsilon_0^2} + \frac{c}{r_0^6}\right) && (9)\\ &= \frac{1}{r_0^6}\left(-\frac{\alpha p^2}{(4\pi)^2\varepsilon_0^2} + \frac{c\alpha p^2}{32c\pi^2\varepsilon_0^2}\right) && (10)\\ &= -\frac{\alpha p^2}{32\pi^2\varepsilon_0^2 r_0^6}. && (11)\end{aligned}$$

Returning to Equation 6, the attractive force has the maximum value at the point where $dF/dr = 0$. Differentiating F in Equation 6 with respect to r, we arrive at Equation 12.

$$\frac{dF}{dr} = \frac{42\alpha p^2}{(4\pi)^2\varepsilon_0^2 r^8} - \frac{156c}{r^{14}} \tag{12}$$

From this the distance where this maximum force occurs is given by Equation 13,

$$r_m^6 = \frac{156c(4\pi)^2\varepsilon_0^2}{42\alpha p^2} = \frac{416c\pi^2\varepsilon_0^2}{7\alpha p^2}. \qquad (\mathbf{13})$$

Then by putting Equation 13 into Equation 6 we have this maximum force in Equation 16.

$$F_m = -\frac{6\alpha p^2}{(4\pi)^2\varepsilon_0^2 r^7} + \frac{12c}{r^{13}} \qquad (\mathbf{14})$$

$$= \frac{1}{r^7}\left(\frac{-6\alpha p^2}{(4\pi)^2\varepsilon_0^2} + \frac{12c(42)\alpha p^2}{156c(4\pi)^2\varepsilon_0^2}\right) \qquad (\mathbf{15})$$

$$= -\frac{9\alpha p^2}{52r^7\pi^2\varepsilon_0^2}. \qquad (\mathbf{16})$$

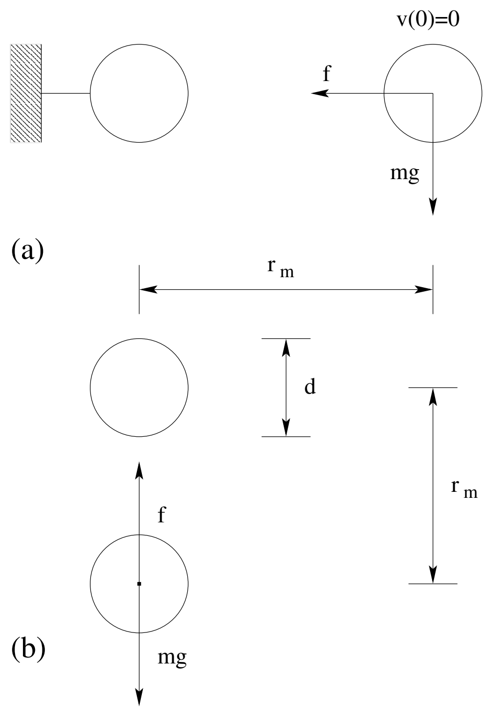


Figure 4 *Force balance with one particle fixed*

Let each of the spherical particles has a radius $r = 1\ \mu$m and the density $\rho = 3{,}000\ \mathrm{kg \cdot m^{-3}}$ (compare the density of silicon, which is

2,329 $\mathrm{kg} \cdot \mathrm{m}^{-3}$). Then the mass of the particle is $m = \rho V = 3000 \times (4/3)\pi \times (10^{-6})^3 = 1.26 \times 10^{-14}$ kg and the weight is $mg = 1.26 \times 10^{-14} \times 9.8 = 1.23 \times 10^{-13}$ N. We may now find the attractive distance, the maximum distance whereby two particles will come together under the van der Waals force. Figure 4 shows the capturing of one particle by another when one particle is fixed in space and another particle has zero velocity but is free to move. Recall that the capture occurs when the acceleration due to the weight of the particle g equals the acceleration a due to f.

If we suppose that our particle have the same polarisability as that of a benzene in its gaseous state, C_6H_6, *i.e.* $\alpha = 11.61 \times 10^{-40}$, then in this case $\alpha = 11.61 \times 10^{-40}\ \mathrm{F}^{-1}\mathrm{m}^4$ (*cf* de Podesta, 1996) and Equation 16 gives us the separating distance between the two particles which gives the maximum attractive van der Waals force, which is 444 μm. Giving our particle other values of α, with the value of α for methanol gas CH_3OH, *i.e.* $\alpha = 3.860 \times 10^{-40}\ \mathrm{F}^{-1}\mathrm{m}^4$, we have this distance $r = 379\ \mu$m, and with $\alpha = 1.647 \times 10^{-40}\ \mathrm{F}^{-1}\mathrm{m}^4$, that of water vapour, the distance becomes $r = 336\ \mu$m.

These values are rather large compared with the radius of the particles, therefore we shall opt instead to a bigger size of particles, when $r = 5\ \mu$m. With the radius of five microns, the particle has a mass of 1.571×10^{-12} kg and its weight becomes 1.64×10^{-11} N. Then if we adopt the value of α for water vapour, $\alpha = 1.647 \times 10^{-40}\ \mathrm{F}^{-1}\mathrm{m}^4$, then from Equation 16 the distance where the force is maximum becomes $r = 167\ \mu$m.

But this is not everything. So far we have only considered what two particles will do when one of them is fixed and the other one has no initial velocity. There are two other things that can happen, the particles may both be moving down beside each other under gravity and the path of the particle which is effected by their mutual attraction. Even when two particles do not come together, their path can be deviated by the van der Waals force. But here for simplicity we shall neglect this and assume that particles either come and stay together or they experience no mutual force whatever. Only if they come together will their final path and velocity be affected, and these depend on their combined momentum.

When particles move relative to each other, their captive velocity depends on their relative velocity. They tend to join each other more easily if their paths are along side each other and goes in the same direction. In the extreme case where both particles move in the same

direction with zero relative velocity, they would come together across a vast gap in between indeed, had there been no frictional force due to viscosity that acts to drag them.

In the present study, the friction due to the fluid in the medium is neglected, together with the relative velocity between the particles, for the purpose of deciding whether particles will come together. It is expected that on average particles coming to within the capturing radius of each other will come together. This radius is larger than the radius of maximum force in Equation 16 above. In other words, we expect our particles to behave like solid spheres with a well defined boundary for their sphere of influence and a much simplified capturing mechanism.

We shall define this radius of captivity, r_c, to be one third that of the radius of maximum force r_m. From Figure 2 this is the point where $r_c = 1.5\sigma$, and because r_m is here 1.2445 we have $r_c/r_m \approx 1.2$. For Figure 2, this is the same as saying that $r_c/r_0 = 1.34$.

One additional point is that, in real situation when particles form a cluster their collective value of the molecular polarisability will change, and this value will not be the same for the cluster as it is for the individual particles. But here we assume that they are the same, which means that the capturing radius of a cluster will be the same as that of the particles which form it.

It may worth mentioning here that the molecular polarisability is related to an optical property, *viz.* the refractive index n_r, by the relationship $\alpha = \varepsilon_0(n_r^2 - 1)/n$, where n is the number density of molecules. For gases $n = P/(k_B T)$, whereas for liquids $n = N_A\rho/m$, m being the mass.

Next we shall concern ourselves with clusters of particles thus formed. All clusters will be assumed to be a closest-conglomerate of particles which form them, with the densest packing density possible in three dimensions, that is $\pi/3\sqrt{2}$ of the face-centred cubic lattice. Numerically this is 0.7405.

Notice that the packing density in two dimensions can be higher than this. Packing circles on a plane is the densest of all with its packing density of $\pi/2\sqrt{3}$.

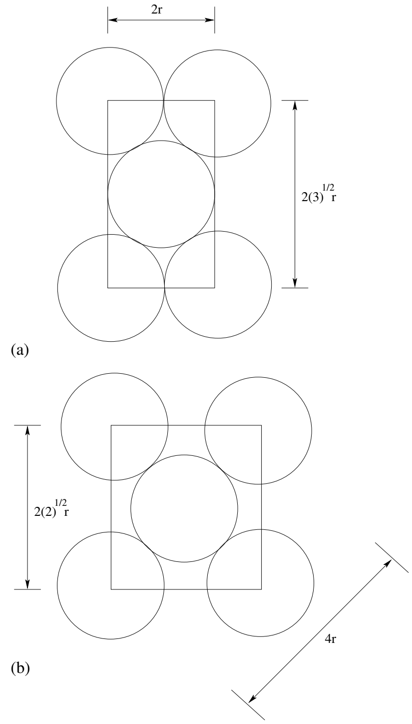


Figure 5 *The packing density calculation of the closest-packed densities*

Shown in Figure 5 are the hexagonal lattice packing in two dimensions and the cubic close pack. In the cubic close packing spheres in every third layer lie vertically straight on top of one another. Each face of its cubic section looks like the second picture in Figure 5. Similar to the cubic close packing is the hexagonal close packing spheres in every alternate layer of which lie over one another. Both the cubic- and the hexagonal close packing have the same packing density which, in the case of the former, is calculated, from the second picture, as $\rho = \sum v_s / \sum v_c$, where v_s are the total volume of sphere segments in the unit cell which has the volume v_c. Here $v_c = (2\sqrt{2}r)^3$ and $\sum v_s = (8(1/8) + 6(1/2))4\pi r^3/3$. In 2 dimensions, the densest packing is calculated from the first picture.:

The density of the packing is $\sum a_c/a_r$, where a_c are the areas of the circles and a_r is the area of a rectangle. These two areas are namely $a_c = 2(\pi r^2)$ and $a_r = 2r(2\sqrt{3}r)$, which give the density $\rho = \pi/2\sqrt{3}$.

If the particle size is very small there will be an effect from quantum mechanics. The de Broglie wavelength, $\lambda = h/p$, exists for all particles large and small. Here $p = mv$, $m = \rho V$ and $V = \pi d^3/6$ for a spherical particle. Figure 6 shows a graph between diameter and wavelength of particles.

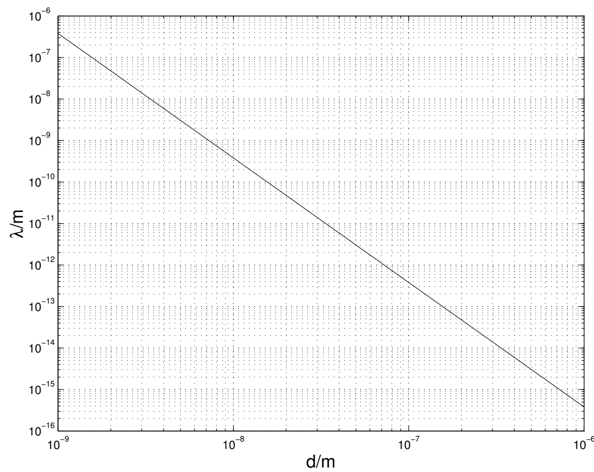


Figure 6 *Quantum-mechanical effect in filtration*

We can see that the quantum-mechanical effect when the pore size is in the order of micrometre will become prominent if the size of particles challenging it is in the order of nanometre. When this is the case there will be not only the interference effect of the particle-wave passing through slits but also the effect of confined particles which produces standing waves of these wave-particles in each pore. These standing waves of the particle-wave have $\lambda_n = 2D/n$.

Bibliography

Michael de Podesta. *Understanding the properties of matter.* 2nd ed. Taylor & Francis. 1996 (2002).

K N Tiyapan. Division of space by Voronoi graphs, percolation within percolation and application to the models of porous membranes. *Ph.D. Thesis.* University of Manchester, 2004

Percolation and Philosophy

Kit Tyabandha, Ph.D.

1st August 2004

Tiyapan wrote the following essay between 2001 and 2003 as a part of his PhD thesis. As it is not directly related to the major works therein I am told it has not been included in the final version of his thesis (Tiyapan, 2004) to appear. Nevertheless some of the ideas he put forward there are new and quite interesting. Therefore I include it here as part of our Essays Series on Percolation. The following is what he had written dated 23rd September 2003 .

Philosophy precedes science, and so the case is with Voronoi tessellation by more than two hundred years. In his philosophical scientific work, Descartes (Des-cartes, 1644) imagines the universe as a tessellation of vortices surrounding stars. According to the pictures he gave, a few of which are duplicated many times over, the positions of these stars are random. Among these, the most abundant picture resembles a two dimensional picture of biological cells or a Voronoi graph. His idea is that stars always position themselves as far apart from one another as possible. At any instance each of these bodies resides within a partition of its own, where it acts as the centre of a vortex around which everything inside the partition revolves.

However, when one would like to track the movement made by a body one may consider the rest as having a fixed position. Starting from anywhere within a partition, the body under consideration moves towards a boundary that lies across its trajectory. Once the boundary is reached, it continues moving along that boundary until it reaches a vertex from the requirement of keeping itself furthest from surrounding stars, and then it chooses the one of the two choices of boundary branches which would make it deviate the least from its present direction. In this way a body snakes its way further and further from its starting position. Although the pictures that Descartes gave were in two dimensions, there is no doubt that he means, and was talking about a three dimensional space. Therefore these two dimensional pictures of his can either be a 2–d section of a three dimensional space, or simply a 2–d representation of it.

As is the case with the study of incompressible fluid flow by vi-

sualising it as a displacement of small rigid spherical particles, whose treatment within the same treatise of his predates the ground-breaking theoretical works of Hele-Shaw (1899) by more than two and a half centuries, the idea he has given about the movement of heavenly bodies predates the idea of van de Weygaert where the similarity is obvious in both the treatment of, as well as the objects considered. Van de Weygaert and Icke (1989) considers the movements of galaxies within voids with sizes several orders larger than themselves. In their simulation, each galaxy moves outwards from anywhere within a void until it reaches a wall where, because of the interaction with other galaxies coming from a neighbouring void which it meets, it loses its radial velocity and continues to move with reduced speed along the plane of the wall. Eventually it will come to an edge where it loses a second part of its velocity components, again thanks to other galaxies this time coming from the other two of the three faces which comes together at the edge. The last component of velocity it has left then takes it as far as the first vertex it should meet and no further. Having lost all the remaining energy to travel further, again through the encounter with other galaxies this time from three other edges which meet with the edge along which it has been travelling, it joins other galaxies to form a cluster which keeps growing all the time.

If the idea of Descartes is that of stars travelling along a Voronoi graph, then the idea of van de Weygaert and Icke represents a universe which is lumpy at Voronoi vertices. Seemingly different, both ideas are very similar. Both consider partitions containing nuclei. The only difference seems to be that in the former case these nuclei are massive bodies, while in the latter they are voids. Thus in a way the two can be said to be dual to each other. Furthermore, the scale being considered seems to be by many orders different. Descartes' vortex idea can easily apply to the scale size of a galaxy, while the vertex idea proposed by van de Weygaert and Icke seems to agree well with observations made in scale unimaginably large compared with that of the other, since they obviously consider giant clusters of over a thousand galaxies in size as being vertices, in other word points.

Voronoi tessellation thus came up philosophically first and then science. Percolation theory, on the other hand, is itself philosophical. Not only that it often sheds light on the underlying reason of many things, but it sometimes is the core philosophy in its own right.

Karma is the central philosophy of Buddhism. In East Asia there are similar sayings which mean *Karma is deed* (*cf Lan. Kamm̱ p̣ẹn kạrkdaṃ* and *Th. Karrm gụe kạrkradaaṃ*). The word *karma* in English adopts a

more practical one among the plethora of implications or nuances in meaning, which says that it is the sum of actions of a person in one of his successive lives, which decides his fate in the next. Another meaning puts it simply as destiny or fate. Karma and enlightenment are the two things around which Buddhism philosophy revolves. Karma entails enlightenment. In other words, the latter is a subset of the former. To put it plainly, enlightenment is percolation while karma is the corresponding percolation process.

To me the teaching of Buddha is about percolation, and hence the idea was discovered by Buddha who is a great philosopher, at least more than two millennia and five centuries before it was rediscovered in Physics of our time. In his book Tiyapan (2003a) talks about himself having become a Christian after having read *Les Misérables* by Victor Hugo. In other works of his (for instance Tiyapan, 2003(b to e)) we learn also how the revelation he experienced gives him an insight and understanding of the New Testament, and how Buddhism, Christianity, Islam and Bhagavad Gītå. say the same thing through the *readers' Me*. Both the conversion and the revelation, however, are a result of many underlying momentums one of which is the college he attended which belongs to the Congregation of the Brothers of Christian Institute of St. Gabriel that was founded in 1705 by Saint Louis Marie Grignion de Montfort (1673–1716).

When we work to glorify God, no doubt at each step forward and as each new idea comes in we are a step closer to our goal. The Internet and plastic money or credit cards may be factors which help the birth of the Euro possible. But the potential and connectivity has already gathered up considerably since the fall of the Iron Curtain and the reunification of Germany with its amazing one to one exchange from the eastern to the western Deutsch Marks. As much as that the fall of both the Iron Curtain and the Berlin Wall may owe much to the Internet and the spread of information by the media the internal flaws within the communist regimes of the Eastern Block countries must have also played a great part. Because otherwise why the same has not happened to the West saturated with the media? The West with its distributed government, a system inherited from America, has an advantage of more robust a system. This is a strong point of democracy. Socialism also has its unique strong point regarding the feeling of security in individuals which helps making the system robust. So as the flaws inside a percolating cluster collectively break it down, another structure is formed up from the ash like a phoenix. As much as individual parts form a single structure, *like brooks make river, rivers run to sea*, so do impurities within a structure break it up the same way as flaws do the

percolating cluster. The spread of terrorism from the middle east owes much to the connectivity of air routes.

If we accept Plato's (360 BC) argument that men are the same because all human souls are the same, then any big difference in the cultures of the world can only be satisfactorily explained by using the percolation theory. If all human souls are the same and souls are more important than bodies, then all men are the same and all societies being comprised of equivalent people will necessarily have an identical structure or structural basis. Intrinsic attributes of a structure, for example the culture of a country or the way of life of its population, can take on various forms according to the phase which it has arrived at through the percolation under the various factors which includes historical and geographical influences. One example is the stark contrast on the value given to winning and losing between the western and the eastern cultures. As I see it, for the former the emphasis is on winners, which is reflected in sayings like *winner takes all*, as contrasted with the emphasis in the latter on losers, for example the Thai saying *win and become the devil, lose and become a saint* [bạ̄e pen bra, jȧna pen mạr]. As another example, let us consider the japanese saying *akirameta yo nanisama akirameta, akiramerarenu to akirameta*, which are seemingly easy but which has eluded some distinguished western experts in the Japanese language and culture (*cf* Tiyapan, 2003a). It was Keiko Saitou who introduced me to this poem in time of trouble. Blyth (1959) translated it as, 'I have resigned myself! Resigned yourself to what? I have resigned myself to never being resigned'. But I think the more likely translation is the following soliloquy, 'You are beaten! In what way do you say I am beaten? If you said that you shall never be defeated then in fact you have already lost'. Kyūdou teaches one that results are nothing. This is not only true for kyūdou., but to all martial arts as well, provided that they are genuine. The training in any art of fighting is never to learn how to fight, but to train the subconscious mind such that it will be able to act by it self at the right time. The subconscious mind is similar to what Freud calls *superego*. In line with Plato's argument that the human souls are never evil, to martial arts this subconscious mind invariably carries the same attributes to that of the superego. In such perspective the Freudian id has no part to play and can be considered as being neither a subconscious mind nor on the same level with the superego; whenever pushed one never push back, and every fight is a self-protective one. The same is the case with kyūdou where there is a difference between an arrow hitting the target and the pride of a successful hit. An arrow hits the target best when it was already there before you took aim. Similar to the idea of complex superposition in the quantum mechanics theory, the free subsconscious mind superimposes and becomes one with the surrounding which includes the target

as well as the path towards it through the air. The arrow thus runs home on its own accord. No one could have put it there; certainly not yourself, so there never is anything to be proud of.

Only a few months earlier it was that I learnt the art of Kyūdou for the first time when on 14^{th} September 1997 in the 36th Meguro-ku Sport Festival of the Meguro district in Tokyo I led our underdog B Team of three to win second. The other two members of the team were Arakawa Hiroshi and Suzuki Jirou. Looking back I can still remember how when one third through our round and having not scored a single point I suddenly feel one with the surrounding, totally oblivion to the sound of the audience behind us, and let go of the target with the result of which not only that I could break the egg but also the way the manner in which the arrow hit the target was perfect. I am proud to say now that even though that hit was the first and only one by me in that event, the other two of my teammates from that point onwards never again failed a single shot through to the end of our round. I wonder if it is not the case that, having been trained to be at ease and relaxed with the bow, my subconscious being, that thing inside me which still do not know much about, kicked in at such a right moment that percolated or communicated itself to my teammates, and then, being sure of having accomplished the task, left the scene. I am rather glad to be left anonymous in this way. To account for my claim of not having consciously aimed that transitional shot, for fifteen years none of my glasses has been adequately prescribed to let me see the bullseye clearly from that distance. Probably nobody will ever know that it was Kit Tiyapan whose name stands first among the three members of the B Team because, reluctant to have my name written in that outlandish katagana appear on three certificates for fear that it disgraces my teammates, I went by the name Zhāng Míng Lóng, what was given to me by my late grandfather and its sole user.

Waking up to the radio on 30^{th} May 2002 morning I heard on the BBC4 one writer as saying that international conflicts and crises occur because matters take on *momentum* which is sometimes difficult to reverse, that it can not be left alone to resolve like playing the russian roulette. I think the only thing that can solve an international, or internal, crisis is a counter momentum built up in the opposite direction. Some says that force stops dangerous mechanisms and that politics never work. No doubt any politics that has no strategies by which to reach at such counter momentum as needed will probably never work.

Life is a dream, or dreams are the other phases of life. If we consider dreams to be in the domains where our minds are in different phases from our awaking selves, then the analysis of the dreams would amount to the study of these various phases. Qualitative sayings like this are sometimes mistakenly written off as nonsensical in the field of science. What is meant when Sacks (1973), for instance, speaks of 'romantic science' being badly needed is that even though art can stand apart from science, science can not be separated from art. I do not think such 'romantic science' is that badly needed, at least not in the field of geometry, mathematics, or percolation. But then again, the first two of these are to me essentially the languages, while the last one the music, and therefore also a kind of language, of nature. Music is the language of the mind and the thing that holds it together. Arts and music heal the mind. But they may not work, which makes them different from medicines. Something mysterious has to trigger before they do. As the *ear* of science they are the things which hold together the *eye* or structure of science.

Opposing forces in nature is an immemorial theme which recurs time and again. This dualism or the law of polarity shows itself, for example, in the eastern philosophy of the opposition between Yin and Yang. This theme came up again in the mid 20^{th} century in the name of percolation, the interplay between two opposing phases. The dynamic triad where the opposing *thesis* and *antithesis* come together and resolve each other into the *synthesis* is analogous with, for instance, the opposition between congested and free-flowing roads in the traffic network which resolve each other in the case where $p_c < 0.5$ into the band of synthesis having the width of $2(p_c - 0.5)$ and centred at 0.5 [Tiyapan, 2004].

The conceptualising power of man culminates in the recurrent themes found more prominently in creative minds (Hankins, 1997). These recurring themes represent *faith* in science. And if we accept that every from of faith is ethical, then all these themes are ethical; in other words, the theme of the antagonistic power between Yin and Yang is an ethical part of the eastern philosophy and the theme of the opposition between phases in percolation is an ethical part of the western science.

Percolation is a twentieth century formulation of an old thing. The last straw that breaks the camel's back is exactly the same as the last jigsaw edge added to a network one step before the onset of percolation (*cf* Tiyapan, 1995.) Andrew Lloyd Webber says the same thing with,

'one Rock *n* Roll too many takes its toll and the soul out of you'.‡ But then again, he is also the product of the twentieth century.

Bibliography

Reginald Horace Blyth. *Oriental Humour.* Hokuseido. Tokyo. 1959.

Renati Des-cartes *Principia philosophiæ.* 1644. Ludovicum Elzevirium. *cf* René Descartes. Œuvres philosophiques. Vol.III. 1973. Garnier Frères. Ferdinard Alquié ed.

Thomas L. Hankins. Triplets and triads. Sir William Rowan Hamilton on the metaphysics of mathematics. *Isis.* **68**, 242, 175–193. 1977.

H. S. Hele-Shaw. The motion of a perfect liquid. *Nature.* Vol. 60, p. 446–451. September 7, 1899.

Plato. [Aristocles] *Phaedo.* [Phaidon] 360 BC.

Oliver Sacks. *Awakenings.* Piccador. Pan Books. 1973(1982).

Kit Tiyapan. *Percolation within percolation and Voronoi Tessellation.* Kittix. Bangkok. 2003a.

—. *A Kiwi Lanna.* Kittix. Bangkok. 2003b.

—. *A British Lanna.* Kittix. Bangkok. 2003c.

—. *Edokko no Lanna.* Kittix. Bangkok. 2003d.

—. *The Siamese Lanna.* Kittix. Bangkok. 2003e.

Kittisak Tiyapan. *Computation of Fluid Flow.* MSc Dissertation. University of Manchester Institute of Science and Technology (UMIST.) 1995.

Kittisak Nui Tiyapan. *Division of space by Voronoi graphs, percolation within percolation and application to the models of porous membranes.* PhD Thesis. UMIST. 2004.

‡ In *The New Starlight Express*.

R. van de Weygaert and V. Icke. Fragmenting the universe II. Voronoi vertices as Abell clusters. *Astronomy and Astrophysics.* **213** 1–9. 1989.

Traffic Modelling

Kit Tyabandha, Ph.D.

Traffic networks

In his paper submitted to the Journal of Statistical Physics Tiyapan (1997) introduces a new idea of considering the development of clusters in both phases at the same time. Applied to the context of traffic network, these phases are namely cars and spaces. Furthermore, because both phases reside in one and the same network, there is a symmetry which divides the probability space into three regions, symmetric with respect to $p = 0.5$. This helps divide the traffic condition into three regions as existing literature in traffic study at the time described, namely free flowing, congested and stand still. In particular, this idea explains the difference between the congested and the stand still states. There has been no reply from the journal.

However, the author of this unfortunate paper has later come across one paper by two mathematicians in the U.S., coincidentally published in the Journal of Statistical Physics, which uses the very idea he laboriously introduced almost exactly four years ago without having received a word from either a reviewer or the editor of that very journal. Thus Gray and Griffeath (2001) ‡ reintroduced the idea of anticars, which are essentially vacant spaces, antigaps and antibonds, which purportedly, greatly helps the derivation of their theories thanks to the symmetry involved. Moreover, one reference made therein (Sipress, 1999; who starts his article with, 'Why is traffic so damn bad?') compares the free flow, synchronised flow and heavy traffic respectively with gas, water and ice, and explains that as the latter's can be explained by phase transitions, so can the former's. But this is just what I have tried to explain back in 1997, that in the case of the gas the spaces had percolated but not the water molecule, in the case of the ice it was vice versa, whereas in the case of the water neither had yet percolated, or both of them had yet to percolate. When explained this way, it becomes clear that the water phase within the air as a network has $p_c < 0.5$. And so the case also must be with *antiwater*, for that matter.

‡ Lawrence Gray and David Griffeath. The ergodic theory of traffic jams. Journal of Statistical Physics. vol. 105, nos. 3/4, 413–452. November 2001.

According to Sipress (*ibid.*), the modelling of traffic is a hot subject in which Nobel prize winners, and cold war physicists alike, are flocking to produce world class scientific papers, which seems to include no Tiyapan. But can plagiarism be excluded?

Bibliography

K Tiyapan. Modelling of traffic congestion. submitted to the Journal of Statistical Physics. 4^{th} November 1997 .

Telescope Project

Kit Tyabandha, Ph.D.

Basic research in science is essential for the development of technology, and therefore is crucial to the future of a country. Amidst all instruments and instrumentations the telescope is perhaps one of the best to give someone a lifelong love of science. This project proposes to build efficient and inexpensive observatories, in both the optical and radio wavelengths, that can be used as prototype models for future construction of observatories within this kingdom and in the neighbouring countries.

There are largely two stages, the first being design and construction of pilot models, while the second the development of these models into a prototype model. In each of these stages there are two separate tasks to be done, one for an optical telescope and the other for a radio telescope. Optical telescopes operate on the range of wavelength of the visible light, that is in the order of 10^{-7} metres. Radio telescopes operate on wavelengths longer than this, to upto the order of a metre. The optical telescope will be of a reflection type. The radio telescope will have a dish similar to, but larger than those used in commercial satellite communication. The pilot models will be for the optical telescope 12 inches in diameter, and for radio telescope the dish 6 metres. Similarly the prototype models will be 20 inches for the reflector and 15 metres for the radio. In order to minimise the cost and to promote self-sufficiency, all the material used will be of local origin and all the design made originally to meet our own local and temperate needs.

The work involves designing and building both telescopes, the mounting together with its electrical and mechanical control systems, and the photographic equipments and instruments for the observation. The mirror used for the objective of the reflector will be made by hand by grinding two pieces of mirror together. The one with a concave surface must then be tested for its optical properties before being coated with silver in a chemical process. There is also the designing tasks for the body of the reflector, the dome to keep it in, as well as for the technical and physical appearance of the radio telescope and the motor drives equipped with a controller for both.

The purpose of this project is to design and implement models of optical and radio telescope, and to use them to develop a prototype

that could be used for installation of observatories at locations within Thailand as well as in our neighbour countries, in order to promote a network of research in science within this region. Our next aim is to patent successful models, before letting people implement them free of charge for the purpose of research in astronomy and space science.

Strategically the work needed is to find a suitable location and secure a permission to use it for installation of the instruments. Then we would seek for permission and build similar facilities for other universities around the country, namely Chiangmai, Khonkaen, Silpakorn and Songkhla University's. Through out the project support and help from sources both local and abroad would be welcome. Future connection may include the Manchester Astronomical Society and Jodrell Bank in England, and the radio array facility in Yamanashi, Japan.

For millennia man has known astronomy. Glass appeared in Egypt around 3500 BC, and lenses have been made since 2000 BC. Hans Lippershey invented the refractory telescope in 1608. A year later Galileo heard about the invention and made one for himself, which he then used it for his famous observations the results of which is well known. Descartes (1637) studied aspherical surfaces and suggested combinations of lenses that correct spherical aberration. Mersenne (1636) proposed a reflecting telescope, but never put it into practice. Christian and Constantine Huygens experimented with lens combinations, and introduced new techniques of making an eyepiece (Smith, 1738). Gregory (1663), Newton (1718) and Cassegrain (1672) introduced different configurations of the reflecting telescope. Since then many a large telescope have been built. Silvering of glass was done by a process invented by Varnish and Mellish (Tallis, 1852). Carl August von Steinheil applied it to astronomical mirrors, as reported in *Augsburger Allgemeine Zeitung* on 24 March 1856. In 1943 studies by A McKeller showed that mirrors coated with aluminium reflected more light than those coated with silver.

Heinrich Hertz discovered radio waves in 1886. Gulielmo Marconi sent wireless communication across Atlantic ocean in 1901. He used square-cone antennas. Karl Guthe Jansky built his Bruce Curtain antenna and found in 1932 cosmic radio waves at 20.5 MHz. Grote Reber (1940's, 1941 and 1944) used a 9 metre parabolic reflector he built to map the sky at the radio wavelength of 2 metres. He realised that a radio telescope acts as a radiometer, which measures temperature of distant region in space that is coupled to the antenna via radiation resistance.

Radio noise from the sun was first detected by J S Hey (1946) in the wavelength of 4–6 metres. Radio telescopes, for example those owned by University of Manchester and Ohio State University, have revealed to us the existence of organic compounds in space (*cf* Blitz and Kutner, 1981), which could give us some clue on the emergence of life on earth. Two atom organic molecules were found in interstellar space in 1937 with the detection of methylidyne (CH) at visible wavelength by the telescope at Mt Wilson; then in 1968 three atoms, water (H_2O), and four atoms, ammonia (NH_3), at 1.3 centimetre wavelength, by 20-foot radio telescope at Hat Creek; and then five atoms, formic acid (HCOOH) and cyano acetylene (HC_3N) at 18.0 centimetre wavelength, by 140-foot radio telescope at National Radio Astronomy Observatory (NRAO).

A radio telescope used in astronomy is a passive remote sensing device. It is shaped as a dish the surface of which is a reflector parabolic in shape. Parameters concerned with its design are the impedance, called *radiation resistance*, and its related *antenna temperature*. The latter is not an inherent property of the antenna, but has parameters which are dependent on the temperature of the region the antenna is looking at, hence the terms 'temperature-measuring' and 'remote-sensing' device.

I proposed to visit Jodrell Bank Observatory in the UK several times over the period starting from 17 May 2006 until 17 November 2006. During the said period I also plan to peruse materials in John Rylands Library to help getting the most out of the visits, for future design and construction of observatories and telescopes. With recommendations from the people at Jodrell Bank, I may visit also other observatories in Europe. I intend to introduce the observatory project to people at these places for future problem-solvings and funds.

At first I had planned to visit England for a few months, and had booked a return ticket for 1 April 2006. Then I was told that I would be asked to leave Mahidol University and thus need not return to Thailand, so I bought myself a single ticket, that is without a returning flight, firstly for 16 April, and then postponed to 16 May 2006. As I have not bought a return ticket, I will need to buy this when I am already in England. I do not know how easy this is going to be, but the six month leave period I am applying for here should be enough to cover all contingencies regarding time.

I do not know why that university was not interested in such useful project as this. The university seems to encourage only researches of trivial importance, instead of basic and fundamental projects. Even though this project may not be said to be basic, it aims towards a com-

plete understanding of the whole system and plants. It tries to produce a technology to build everything locally from scratch.

My Observatory Project had been submitted for consideration for Thailand Research Fund. Because of my lacks of experience, it is possible that I may not be successful with my application. But even if this were to be so, I will keep looking for funds both local and abroad. The project is now at the beginning of a designing stage, and therefore still does not need much money.

Costs for this project include those for the design and construction of a prototype and a pilot model telescope. The costs for each model includes the instrument, body, mounting, housing to keep it in, and electrical and mechanical controlling devices.

Bibliography:

L Blitz and M Kutner. Extragalactic molecules. *Proceedings of a workshop held at the NRAO at Green Bank.* West Virginia, 2–4 Nov 1981

René Descartes. *Dioptrique.* 1637

James Gregory. *Optica promonta.* 1663

Karl Guthe Jansky. Directional studies of atmospherics at high frequencies. *Proceedings of the Institute of Radio Engineerings.* Oct 1932

Henry C King. *The history of the telescope.* Charles Griffin & Co, 1955 (Dover, 1979)

John D Kraus. *Antennas.* 2nd ed, McGraw-Hill, 1988

Benjamin K Malphrus. *The history of radio astronomy and the National Radio Astronomy Observatory.* Krieger, 1996

Marin Mersenne. *l'Harmonie Universelle.* 1636

Nature. **157**, 296, 1946

Isaac Newton. *Opticks.* 1718

Philosophical Transactions. **7**, 4057–9, 1672

Grote Reber. Cosmic static. *Proceedings of the Institute of Radio Engineers.* Feb 1940

– Cosmic static. *Astrophysical Journal.* June 1940, and 1941

– Radio emissions from the Milky Way. *Astrophysical Journal.* **100**, 279–87, 1944

Robert Smith. *A compleat system of opticks.* **iii**, 281–301, 1738

Tallis. *History and description of the crystal palace.* 1852

Tessellation

Kit Tyabandha, Ph.D.

Tessellation is a division of space completely into partitions, which can be done in various manners, for example regular, semi-regular and random. It is a generalisation of tilings, which have existed since early civilisations. It is related to the areas of network- and graph theories and, particularly for random tessellations, to that of topology. A good introduction of regular tessellation has been written by Grünbaum *et al* (1987). For random tessellation works were notably pioneered by Voronoi (1908 and 1909). A Voronoi tessellation is a set of regions Π_i covering the space such that $\Pi_i = \{x|\mathrm{d}(x, x_i) < \mathrm{d}(x, x_j)\,\forall j \neq i\}$, where x_i, $i \in \mathrm{Z}^+$, are the coordinates of the nuclei, and $\mathrm{d}(x, y)$ is the Euclidean distance between x and y. This work is in a surveying stage to cover as large ground as possible. Later there will be investigation and in depth study of some specific, well-defined area.

Bibliography

Branko Grünbaum and G C Shepherd. *Tilings and patterns.* W H Freeman, NY. 1987

Georges Voronoï. Nouvelles applications des paramètres continus à la théorie des formes quadratiques. Premier mémoire. Sur quelques propriétés des formes quadratiques positives parfaites. *Journal für die reine und angewandte Mathematik.* **133**, 97–178. 1908

G F Voronoï. Nouvelles applications des paramètres continus à la théorie des formes quadratiques. Deuxième mémoire. *Journal für die reine und angewandte Mathematik.* **134**, 198–287. 1908

G F Voronoï. Nouvelles applications des paramètres continus à la théorie des formes quadratiques. Deuxième mémoire, Seconde partie. *Journal für die reine und angewandte Mathematik.* **136**, 67–181. 1909

The UK Trip

Kit Tyabandha, Ph.D.

The United Kingdom (UK) has been a source of inspiration and a working ground of many a great work. It is also a place where I studied and where I find much resource and connection for my research. I have been in the UK since 16 years ago, but this is the first time ever for me for the purpose of enhancing my research career and networks.

In France, I had spent one month at the University of Paris I at Sorborne/Pantheon, doing mathematics in economics. So there is a possibility that I may be able to spend some time there by crossing over from the UK.

I have a project for designing and implementing observatories for countries this region. The project is called the *Prototype observatory for future of the country in science*.

Great Britain has the Multielement Radio Linked Interferometer Network (MERLIN), which has its centre at Jodrell Bank Observatory at Macclesfield in Cheshire the owner of facilities of which is University of Manchester (UM) to which I am attached as an alumnus. My connection and experience at the facility would definitely increase the possibility of success for my project.

One of my projects has to do with isomorphism in language, which includes natural languages. Not many researchers to my best knowledge, work in precisely this area. But of other works closely relevant to it, the UK is among the leaders and pioneers. It will be helpful for my research if I could spend some time at the School of Oriental and African Studies (SOAS) in London. Several links in my connections already lead me there.

I planned to visit Professor Emeritus David Bell at the department of mathematics, University of Manchester. I also planned to use the archives at John Rylands University Library of Manchester (JRULM) for my research on history of mathematics, in particular works from the 19^{th} century on positive quadratic form, as well as works from earlier centuries leading towards it

Apart from that, a visit the Astronomy and Astrophysics group at the School of Physics and Astronomy at UM could be useful. They not only own Jodrell Bank, but also is a part of the world-wide Very Long Baseline Interferometry (VLBI), a network expanding across the globe which links radio telescope together. Ideally speaking, my Observatory Prototype for South-east Asia (OPSA) project should link this region with this VLBI network. I have had for over ten years from a controlling body (*KV*) a certificate for being a qualified communication engineer, but have never used it. Projects like this OPSA, in a way, lets me use the qualification to do something towards a good cause.

Visit Professor Emeritus Graham Davies, who was the supervisor for my PhD, at the Corrosion and Protection Centre Industrial Services in the University of Manchester. I am interested in projects related to Chemical Engineering and geometry, for example the spatial distribution oil in reservoirs, and the same of uranium deposits in copper ore.

I plan to travel to Jodrell Bank in Cheshire and spend some time at the observatory there. The place is approximately one hour's drive from Manchester.

I try to be in London for approximately one month, to visit SOAS and other places, as well as to read at the British Library, which contains several interesting items that are unavailable at JRULM. I also plan to visit Professor George Rowlands at the Physics Department, University of Warwick, possibly to spend some time with the astronomy and mathematics group there.

Covering Lattice

Kit Tyabandha, Ph.D.

A covering lattice of any two-dimensional lattice is the lattice obtained by joining midpoints of consecutive edges together. The code (Tiyapan, 2004) finds covering lattices up to the eighth one and computes the total area of the cells for each case.

The square lattice is the only regular covering lattice, that is it is both the dual and the covering lattices of itself. But all polygonal tilings and tessellations can have a covering lattice, or in fact an infinite orders of covering lattices.

Coverings can be generalised to a general dimension d. In two dimensions they are lines, *i.e.* having two vertices, straight lines each of which join two lines across a corner. For three-dimensional polyhedral tessellations they are planes with three vertices, triangles each of which join three planes across a corner, in other words a coign. Then in four dimensions they may be polyhedra with four vertices, tetrahedra each of which joins four 4-d polytopes across a four-edged corner in four dimensions, and so on. In d dimensions, then, perhaps they are polytopes with d vertices, three-cornered $(d-1)$-polytopes each of which joins d d-dimensional polytopes across a d-edged corner in d-dimensions.

One interesting property of covering lattices is that they leave the voids intact while reducing only their size. Thus the structure and complexity of the original tessellation remain unchanged. This can be useful when we want to exclude some of the volume. In filtering membrane studies, for instance, this is ideal since all the voids still remain in the same position.

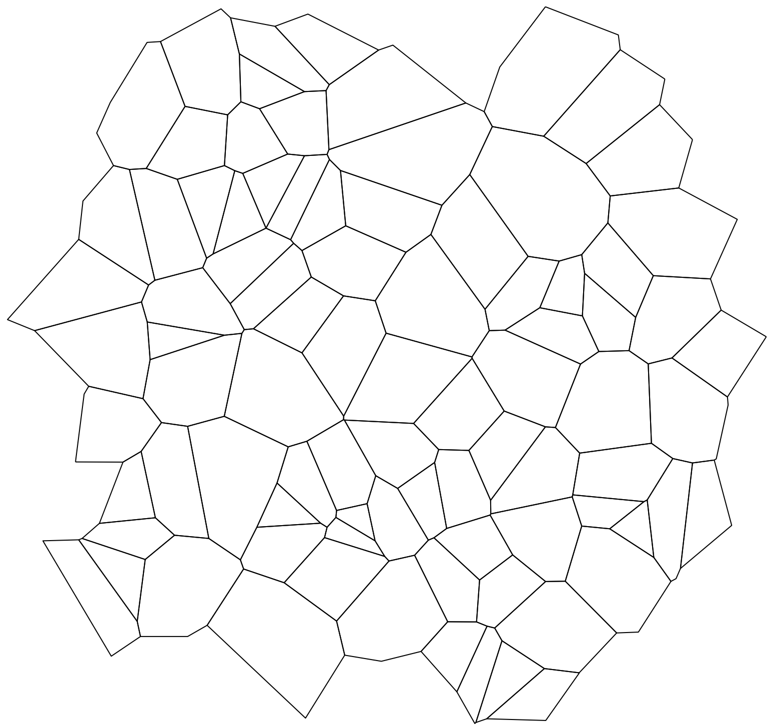
The process of covering is similar to that of shrinkage in cell in that there is a retreat away from corners. This could be because corners are hard to maintain. Surface tension is high there, and like a nook, recess, or remote, unstrategic parts of a country the cost of maintaining and governing is high. In the case of a country, conflict in such parts is analogous to the high surface tension in corners of cells.

When an empire or a metropolitan city declines, the process is similar. Such far corners are the first parts to fall into chaos. Law,

order and security shrink away from them. The Roman Empire is an excellent example of this. In its heyday it reached out to every corner of Europe, however far. When it came to the decline, it literally *pulled itself together*, though it never could pull itself together again after that. It drew away first from those far corners, England for instance, and then towards its nucleus and died.

Manchester is another interesting example. After the industrial revolution, and under the governance of the Conservative party, the city declined. And as it did, all the different nuclei became prominent, if only because the distance between them became more so. Thus Bolton, Altrincham and Stockport, for example, shrunk towards their nuclei, leaving behind dangerous districts where mugging, murder and crime are rampant like the Moss Side decades ago until shortly after the IRA bombing of the city centre. When an urban area fades away it does not do so suddenly but like the plant cell subject to a dewatering process or a polygonal tessellation to a covering one.

Random fluctuation can create areas of irregularity within a homogeneous and isotropic universe. These irregularities become nuclei, and from duality of the structure fine partitions start to develop around them which become Voronoi facets. Gradually but steadily the gas shrinks to form the galaxies of our present universe.



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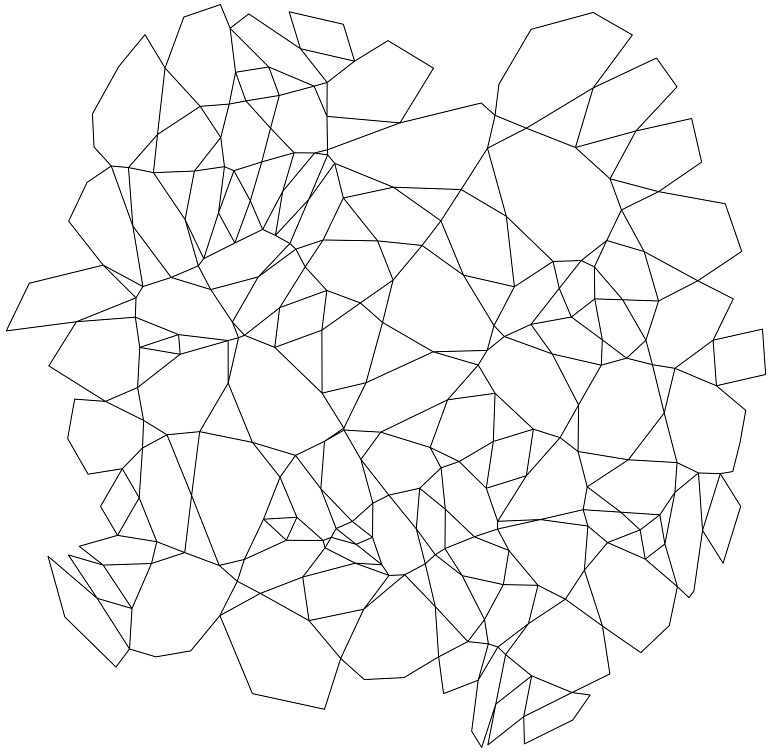

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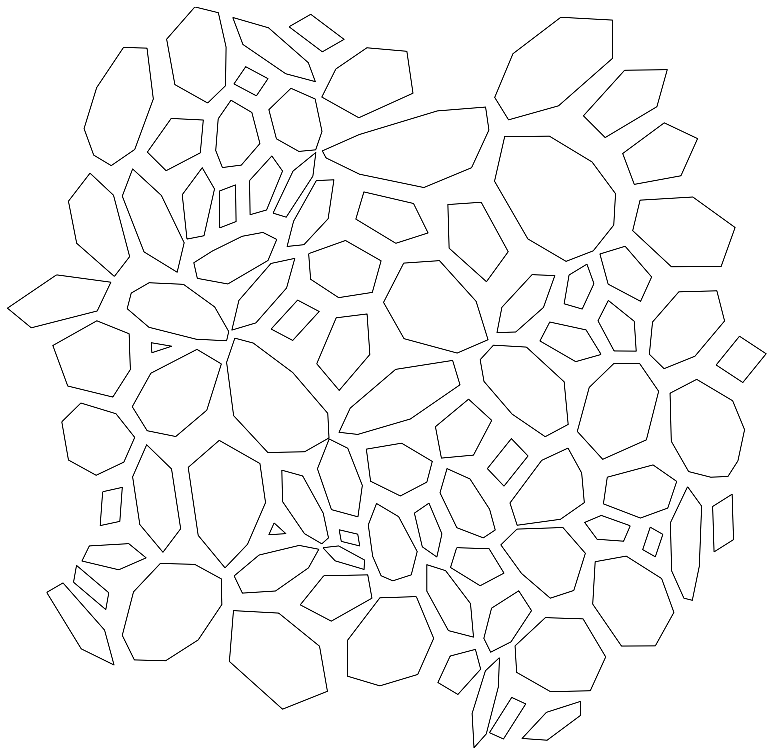

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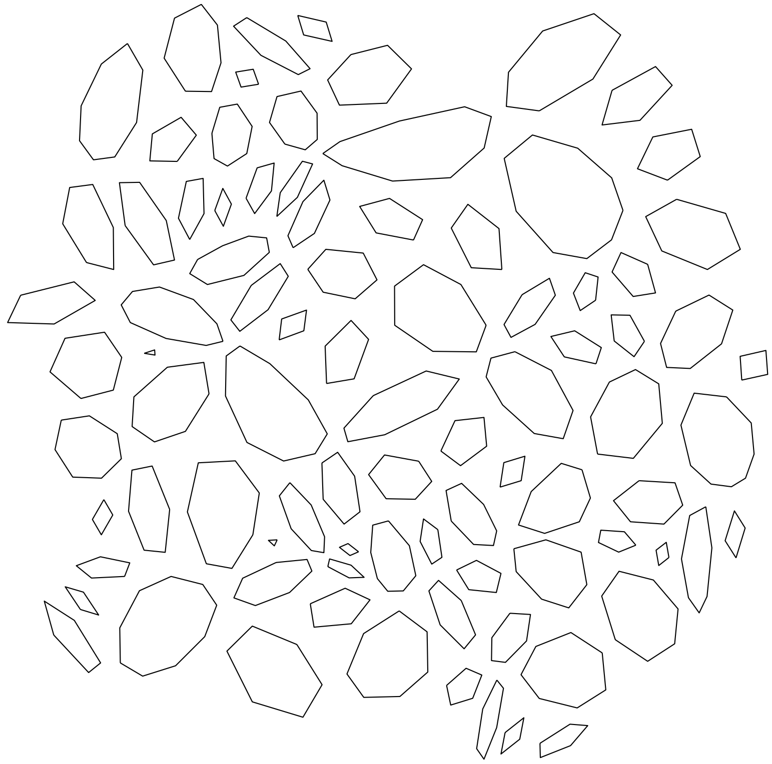

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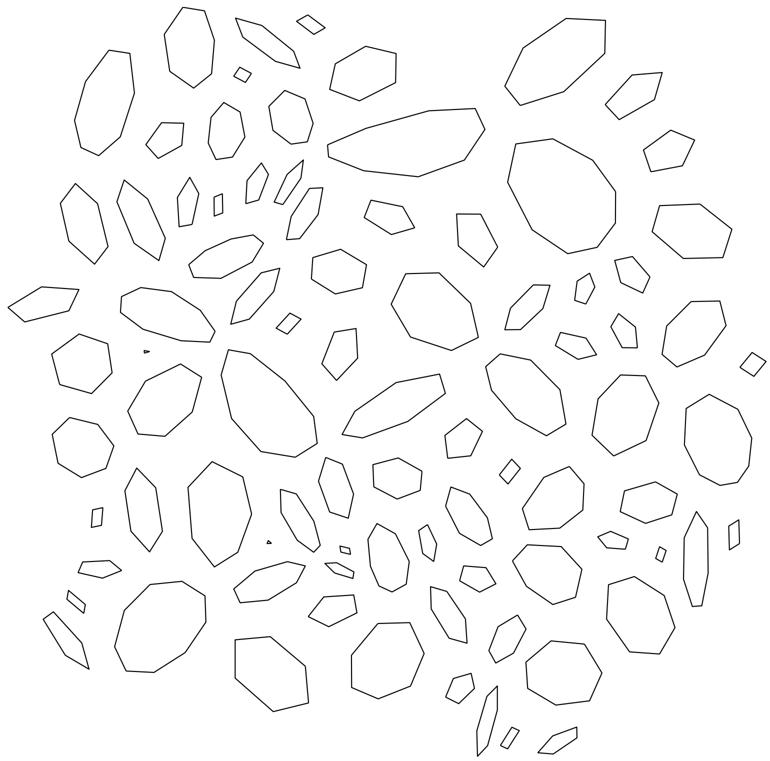

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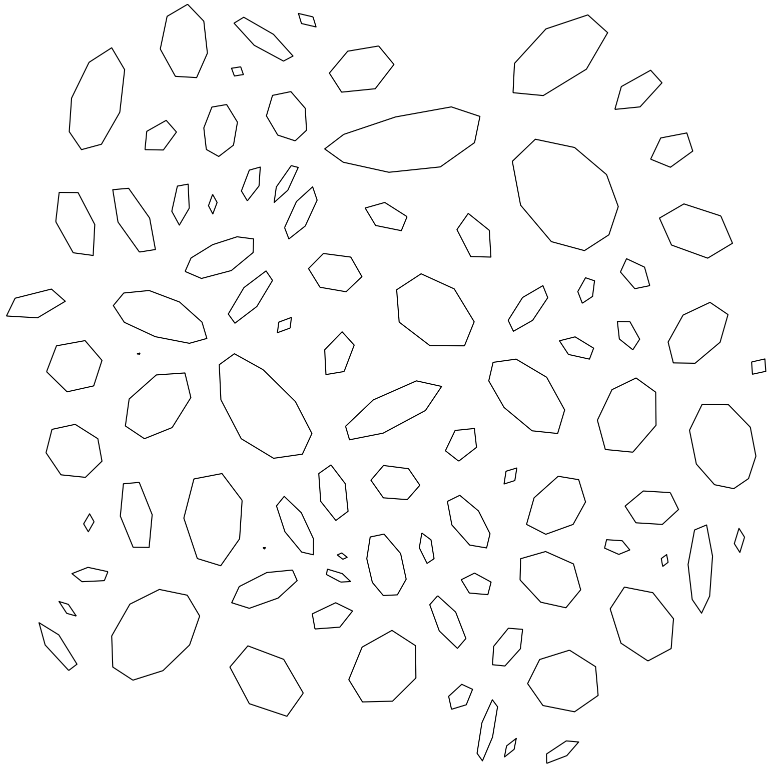

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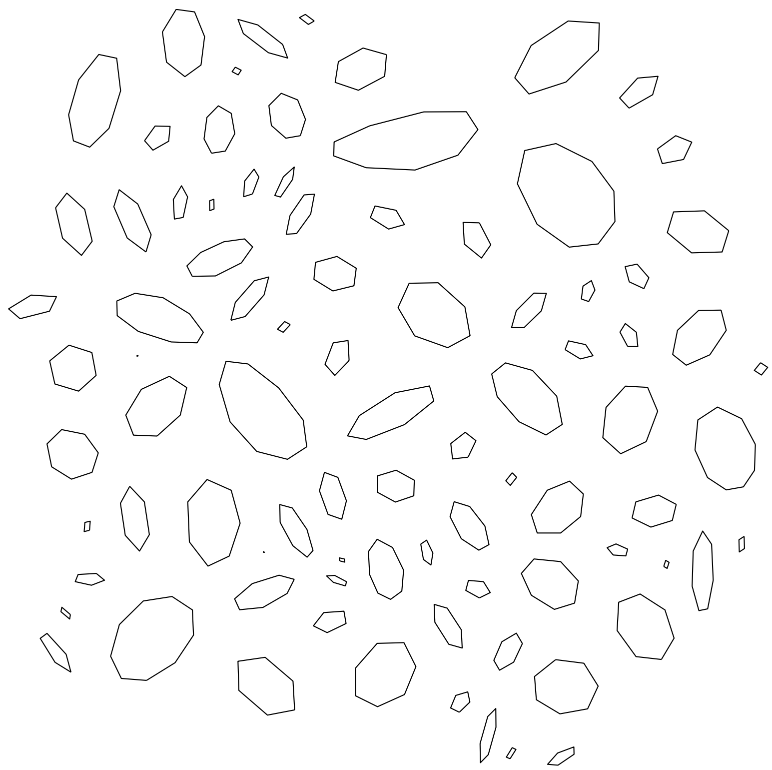

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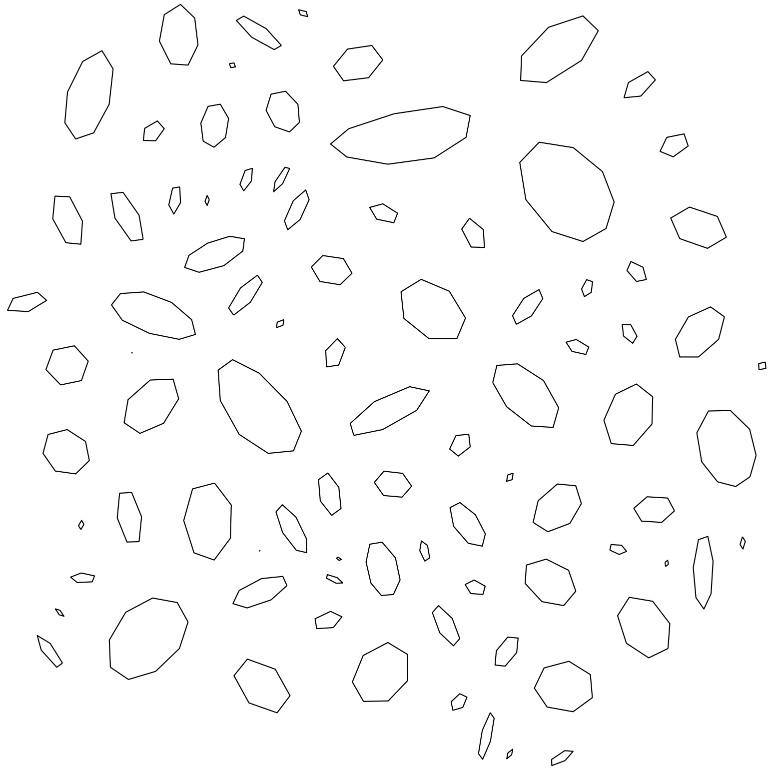

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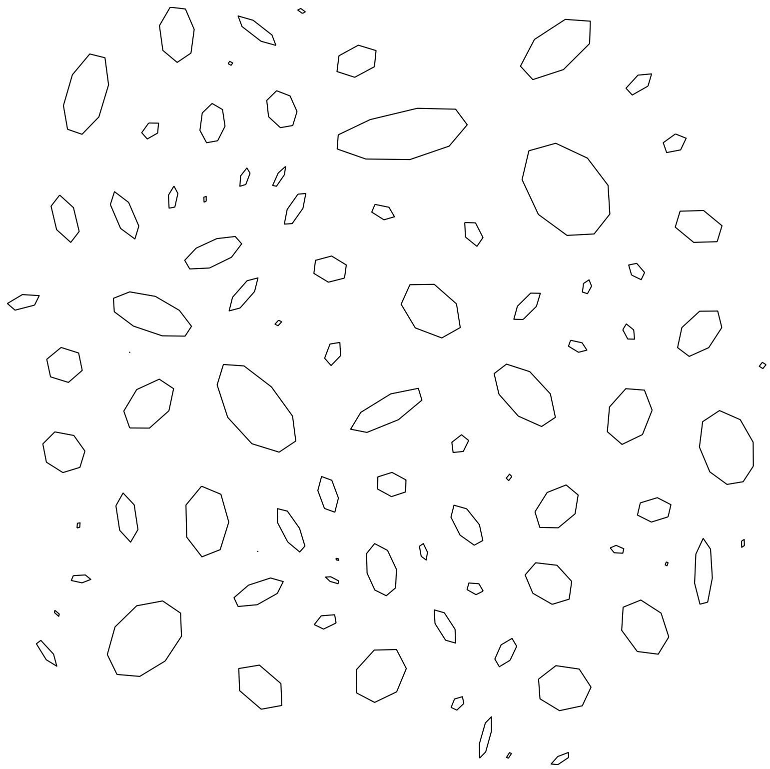

(i)

Figure 1 *Covering lattices, stone pavements, or galaxies in the forming?* (a) $\mathcal{V}$, (b) $C^1(\mathcal{V})$, (c) $C^2(\mathcal{V})$, (d) $C^3(\mathcal{V})$, (e) $C^4(\mathcal{V})$, (f) $C^5(\mathcal{V})$, (g) $C^6(\mathcal{V})$, (h) $C^7(\mathcal{V})$, (i) $C^8(\mathcal{V})$.

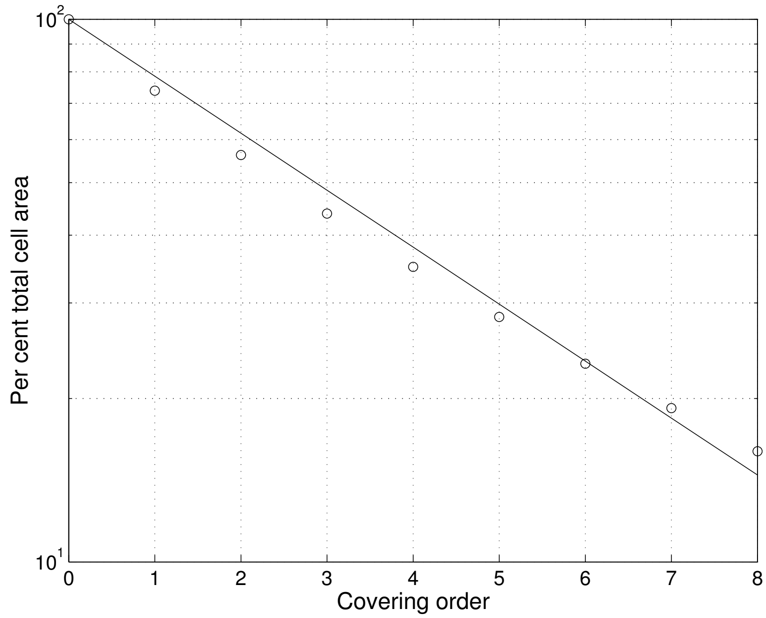

Figure 2 *The area of multiply covered cells*

Figure 1 (a) to (i), which are the results of the covering operator applied to a Voronoi graph eight times in succession, represent this situation. Figure 2 shows the area of multiply covered cells $\mathrm{C}^n(\mathcal{V})$, $n =$ 0 to 8, where $\mathrm{C}^0(\mathcal{V})$ is the Voronoi graph $\mathcal{V}$. Circles are the per cent total area, and the curve is $y = 10^{-0.8x+2}$. The code `gxy.m` to find the covering contours above is given elsewhere (Tiyapan, 2004).. The area of the cells decreases from the covering operator, not linearly but with deceleration as Figure 2 shows.

On taking a closer look at Figure 1, one can interestingly notice that even though all pores shrink from the application of the covering operator, some does so much quicker than others. In particular, round pores shrink but slower. The more corners a pore has the less acute are the angles, which makes it the more stable and thus able to maintain its original size.

Geometrically, one can see that the most unfortunate of all polygons is the triangle. The area of a triangles of any shape reduces by 75 per cent upon being covered. The circle is the most fortunate in this matter since it has no corners and therefore it is impossible for these to be cut. This is in accordance with our argument that corners are

unstable region.

Bibliography

K N Tiyapan. Division of space by Voronoi graphs, percolation within percolation and application to the models of porous membranes. *PhD Thesis.* University of Manchester, 2004